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TECHNICAL REPORT NO. 2

Contract No. AF 30(602)-2452

Prepared for

Rome Air Development Center
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MICROWAVE PHYSICS LABORATORY

GENERAL TELEPHONE AND ELECTRONICS LABORATORIES, INC.
GENERAL
SYSTEM

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THE ABSORPTION OF MICROWAVE RADIATION IN A PLASMA WHOSE ELECTRON DENSITY VARIES LINEARLY WITH DISTANCE

ABSTRACT

The absorption of microwave radiation normally incident on a semi-infinite plasma whose density varies linearly from zero to infinity is calculated for a wide range of density gradients and electron collision frequencies, and represented by a single family of curves. This family can be reduced to a single curve which represents the absorption with good accuracy as a function of the single parameter (z_c/λ_o) (ν/ω) in the range $z_c/\lambda_o > 0.5$ where z_c is the distance from the plasma boundary to the cut-off point λ_o - the free space wavelength, ν - the collision frequency, and ω - the operating frequency (in radian units).

In order to test applicability of this data to more general types of plasma profiles, the absorption was computed for various configurations in which the semi-infinite plasma was modified by a variety of boundary conditions at various distances from the cut-off point. The results indicate that the absorption depends most strongly on conditions near cut-off and to a much lesser degree on conditions farther away. The data calculated for the semi-infinite plasma is therefore applicable to other types of plasma whose density varies smoothly from zero to infinity.

To obtain the absorption in a given non-uniform plasma, one determines the slope of the electron density, dn/dz, and ν/ω, both values being taken at the region near cut-off. One then uses the absorption value as calculated for a linearly varying plasma characterized by the same parameters. The accuracy of this procedure will vary with circumstances but may be considered adequate in many practical cases where experimental parameters are only very crudely measurable.

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ILLUSTRATIONS

I. INTRODUCTION

The calculations presented here are designed to provide approximate data on the absorption of microwave radiation which is normally incident on a plasma boundary for the case where the electron density in the interior of the plasma exceeds the so-called cut-off value.

The electron density in a plasma is conveniently characterized by the plasma frequency ω_p, defined by

$$\omega_p = (e^2 n/m\epsilon_o)^{1/2} \tag{1}$$

where n is the number of electrons per unit volume (henceforth referred to simply as the plasma density), e - the electron charge, m - the electron mass and ϵ_o - the dielectric constant of free space. $\omega = \omega_p$ is called the cut-off point and plasmas for which $\omega_p > \omega$ are referred to as beyond cut-off for the given frequency as they will support only waves of the evanescent type which decay exponentially with distance.

We wish to study the absorption of microwave radiation incident on a plasma whose density in the interior exceeds the cut-off value. If we were to assume a uniform, sharply bounded plasma, we would find that except for a small amount of absorption in the boundary layer almost the entire radiation is reflected. The situation is drastically altered if instead of a sharply defined boundary there is a gradual tapering of the density from its maximum value to zero. Strong absorption then occurs in the

region where ω_p (which is a function of the density n and therefore of position) is comparable to ω. The more gradual the slope of the density, the larger this absorption. In almost all practical cases, e.g., plasmas formed in containers or in shock waves, the density of the plasma is in fact tapered to a greater or lesser degree and the character of this taper will determine the amount of power absorbed.

Our problem is thus to determine the absorption of microwave radiation in a plasma whose density rises gradually from zero to a value beyond cut-off. Two theoretical approaches are possible. The first assumes a particular functional dependence of the density n on position and proceeds to solve the problem numerically. Since, in most practical cases, the plasma parameters are neither well known nor accurately reproducible this method is of little general value. An alternative approach is to treat the problem approximately but in a more general way. The most common of these approaches is the WKB approximation which applies as long as the changes in the medium are sufficiently slow that one may ignore any reflections associated with the non-uniformity. In the neighborhood of $\omega = \omega_p$ the approximation breaks down, and in order to carry the solutions beyond this point, one customarily resorts to complicated connection formulas which are obtained by approximating the variation of n in the neighborhood of the cut-off point by a linear variation. The solution in this linear region is then matched to the WKB solution in the region $\omega_p < \omega$ giving rise there to a reflected as well as to

an incident wave. Physically, this implies the assumption that the reflection process is confined entirely to the cut-off region. The amount of labor involved in this procedure is quite formidable, and it is difficult to display the results of such calculations in a generally applicable form.

In view of the crudeness of experimental data, there is need for a simple criterion or set of tables from which one may determine rapidly a rough value for the amount of absorption in any given practical situation. To accomplish this we carry the WKB point of view a step further by applying the following arguments:

(a) Reflection and absorption of the radiation are determined primarily by the medium properties in the neighborhood of cut-off, since this is the region of strong interaction with the plasma.

(b) The behavior of the medium can be characterized by the gradient of n at the cut-off point.

(c) The plasma can be approximated by one with a density which varies linearly with distance.

The first part of this work consists of a comprehensive tabulation of absorption in a plasma whose density varies linearly from $n = 0$ at $z = 0$ to $n = \infty$ at $z = \infty$*. The radiation is incident along the z direction. The fraction of absorbed power is tabulated as a function of the density gradient and of the parameter ν/ω where ν is the electron collision frequency.

*Fig. 1, Case I.

The rest of the work is devoted to establishing the limits under which the results obtained in the above case can be applied to more general density profiles. This is accomplished by introducing modifications at boundary points in front or behind cut-off as shown in Fig. 1. Three cases are considered. Case I represents the semi-infinite case described above. Case II represents modification in the near boundary obtained by substituting a plasma of uniform density n_1 from $z = 0$, to $z = z_1$ ($z_1 < z_c$ where z_c is the coordinate of the cut-off point). Two choices are made for the value of n_1. In IIa, n_1 is such as to make n continuous at $z = z_1$. In IIb, $n_1 = 0$. Case III represents modifications in the far boundary obtained by substituting a uniform plasma of density of n_2 from a point $z = z_2$ ($z_2 > z_c$) to $z = \infty$. Three values are chosen for n_2. In IIIa, $n_2 = \infty$. In IIIb, n_2 is such as to make n continuous at z_2. In IIIc, $n_3 = 0$.

The above modification presents extreme limits of concave or convex variations of n with distance, and encompass between their limits some of the smoother profile encountered in practice. Deviation of the absorption from that given by Case I becomes more severe as z_1 and z_2 approach z_c. The data obtained therefore indicates to what extent and within what limit the absorption depends on the characteristics of the mediums away from the cut-off region. It therefore serves as a check on the applicability of the data obtained in Case I to more general cases.

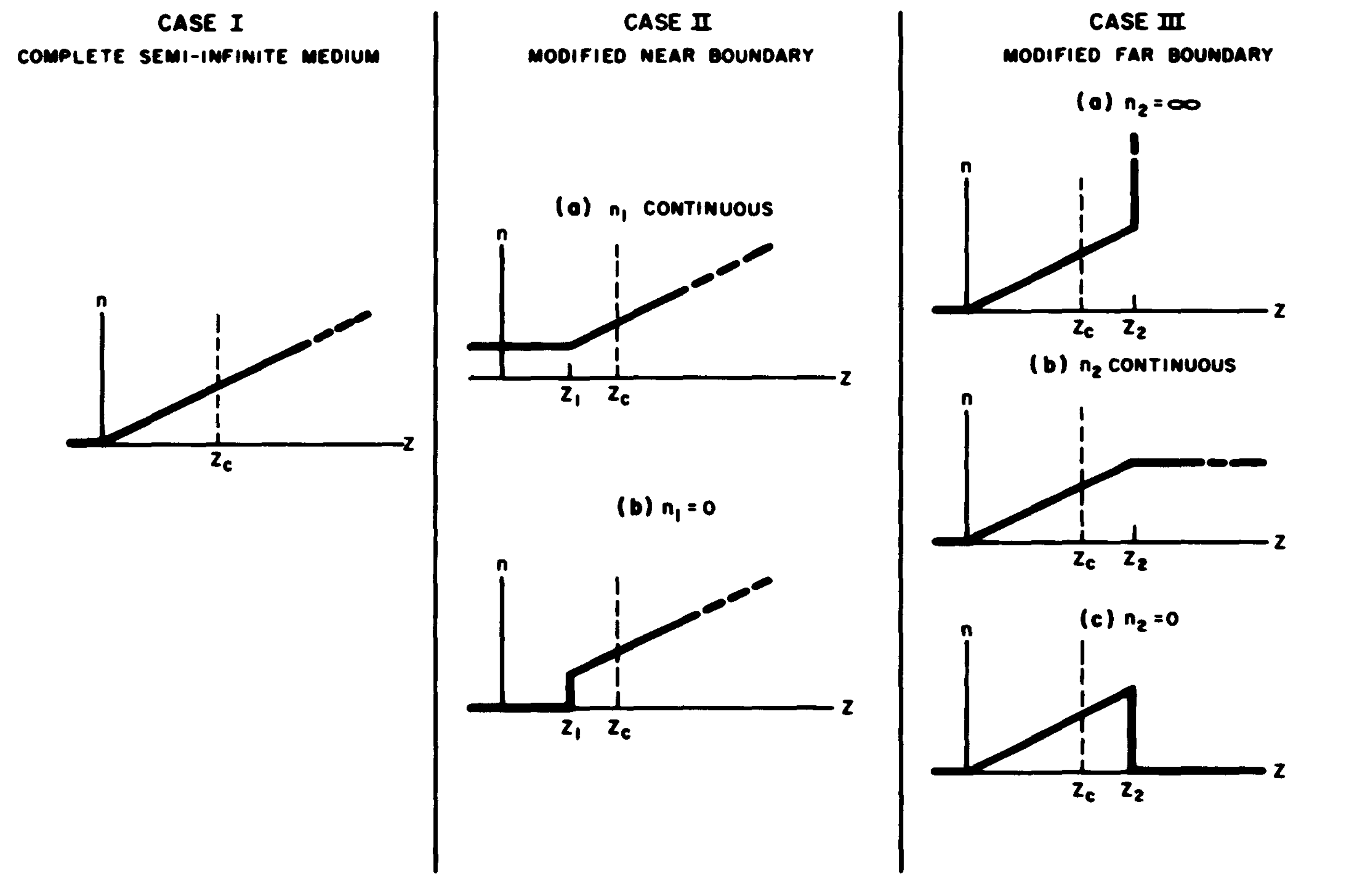

Figure 1 Schematic presentation of the plasma configurations for which absorption is computed. Case I represents a semi-infinite medium with n rising linearly from 0 to ∞. In Case II, a uniform plasma of density n_1 is substituted in the region $z < z_1$, and in Case III a uniform plasma of density n_2 is substituted in the region $z > z_2$. The origin of coordinates is chosen so as to make n proportional to z in the linear section in all three cases.

II. SUMMARY OF THEORY

It is assumed that a plane wave at frequency ω is incident along the z direction on a plasma whose density n is a linear function of z alone. The equation for such a wave is derived in the usual way from Maxwell's Equations and is given by

$$\frac{d^2E}{dz^2} + \omega^2\mu_o\epsilon E = 0 \tag{2}$$

where E is the electric field vector and ϵ, the "effective dielectric constant" is given by

$$\epsilon = \epsilon_o \left[1 - \omega_p^2/\omega^2(1 + i\nu/\omega)\right] \tag{3}$$

According to (1), ω_p^2 is proportional to n. Therefore, if n is assumed to be proportional to the coordinate z, one can put

$$\omega_p^2 = \omega^2 \frac{z}{z_c} \tag{4}$$

where z_c is the coordinate of the cut-off point defined by $\omega_p(z_c) = \omega$.

It is convenient to measure distances in units of the free space wavelength λ_o given by

$$\lambda_o = \frac{2\pi}{\omega(\epsilon_o\mu_o)^{1/2}} \tag{5}$$

and define a new variable

$$u = z/\lambda_o \ . \tag{6}$$

If (3), (4), (5), and (6) are successively substituted in (2), one obtains the equation

$$\frac{d^2E}{du^2} + 4\pi^2 \left[1 - \frac{u}{(z_c/\lambda_o)\ (1 + i\nu/\omega)} \right] \qquad (7)$$

This equation depends only on two parameters, namely z_c/λ_o which measures the distance in which the plasma builds up to the cut-off value as measured in free-space wavelengths, and ν/ω, the ratio of the collision frequency to the operating frequency. Eq. (7) is of the form

$$\frac{d^2E}{du^2} + (Au + B)\ E = 0 \qquad (8)$$

which has the general solution

$$E = a\ Ai \left[- \frac{Au + B}{A^{2/3}} \right] + b\ Bi \left[- \frac{Au + B}{A^{2/3}} \right], \qquad (9)$$

where Ai and Bi are the two types of Airy functions[1] and a and b are suitable constants. It can be shown that Ai and Bi, respectively, play roles analogous to those of forward and backward waves. A complete solution is obtained by applying at the boundaries the standard continuity requirements to E and dE/dz. In Cases I and II, (See Fig. 1), the solution inside the plasma is composed of the Ai function alone. This solution is coupled to an incident and a reflected wave in the region to the left of z_1. In Case III, the solution inside the plasma is a combination of Ai and Bi functions and there is, in addition, a transmitted wave propagated to the right of z_2.

The total power absorbed in the linear plasma region is obtained by subtracting in the usual way the reflected and (where applicable) the transmitted power from the incident power.

Absorption as a function of distance is given by Poynting's Theorem as

$$W = - dP/dz \qquad , \tag{10}$$

where P is the Poynting vector. We shall put W in the normalized form

$$W = - \frac{1}{P_1} \quad \frac{dP}{du} = - \frac{1}{P_1} \quad \frac{dP}{d(z/\lambda_o)} \tag{11}$$

where P_1 is the Poynting vector at z_1. In this form, W measures the relative power absorbed per free-space wavelength. The form of Eq. (11) gives rise to a display of the absorption curves which is more uniform than that provided by Eq. (10).

III. DESCRIPTION OF DATA

In this section we will describe the computed data and its mode of presentation. In each case we list the parameter values for which the data was computed. It will be observed that the results are not displayed for all possible combinations of the parameters. Data which does not differ substantially from that obtained for neighboring values is omitted in many cases.

The data displays the relative total absorption in the plasma, which is defined as the ratio of the total absorbed power

in the linearly varying section of the plasma to the power incident at z_1. Also displayed for Case I alone is the absorption W as a function of distance z as defined in Eq. (11).

CASE I:

a. Total Absorption

Total absorption was computed for all combinations of the values:

ν/ω = 0.01, 0.016, 0.025, 0.04, 0.063, 0.1, 0.16, 0.25, 0.4, 0.63, 1, 1.6, 2.5, 4, 6.3

z_c/λ_o = 0.025, 0.04, 0.063, 0.1, 0.16, 0.25, 0.4, 0.63, 1, 1.6, 2.5, 4, 6.3, 10, 16

These values were chosen so as to divide each decade into 5 parts roughly equal on a logarithmic scale. The data is presented in Figs. 2, 3, and 4.

b. Absorption as Function of Distance

W was computed for all combinations of the values:

ν/ω = 0.01, 0.025, 0.063, 0.16, 0.4, 1, 2.5, 6.3

z_c/λ_o = 0.025, 0.063, 0.16, 0.4, 1, 2.5, 6.3

The data is displayed in Figs. 5a-5g. Each figure corresponds to one particular value of z_c/λ_o, and includes curves representing various values of ν/ω. Only those curves which differ appreciably from curves corresponding to neighboring values are included.

The abcissa is divided in terms of free-space wavelength units. A vertical line indicates the cut-off position.

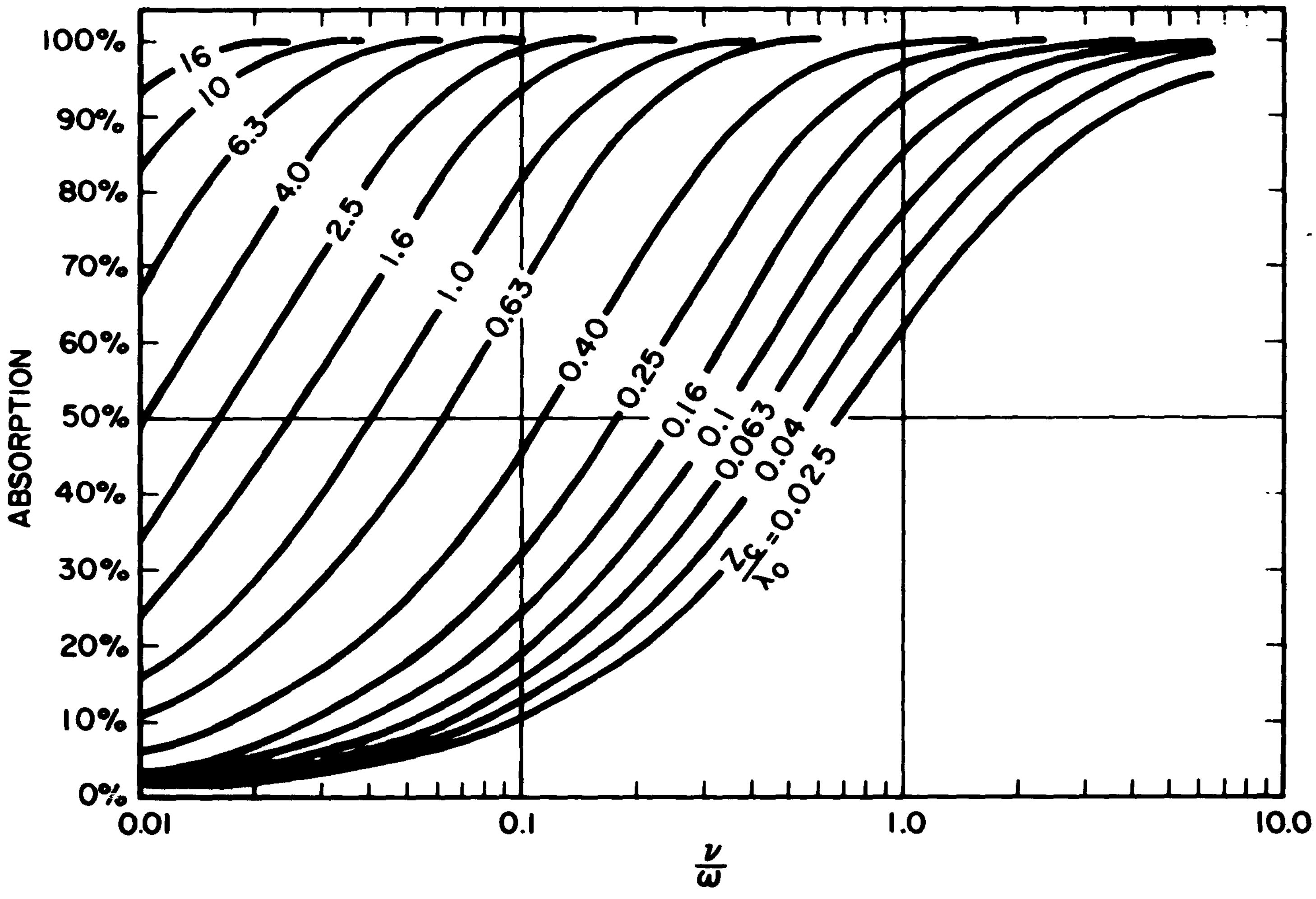

Figure 2 Total absorption in semi-infinite linearly varying plasma (Case I) as function of ν/ω and z_c/λ_o.

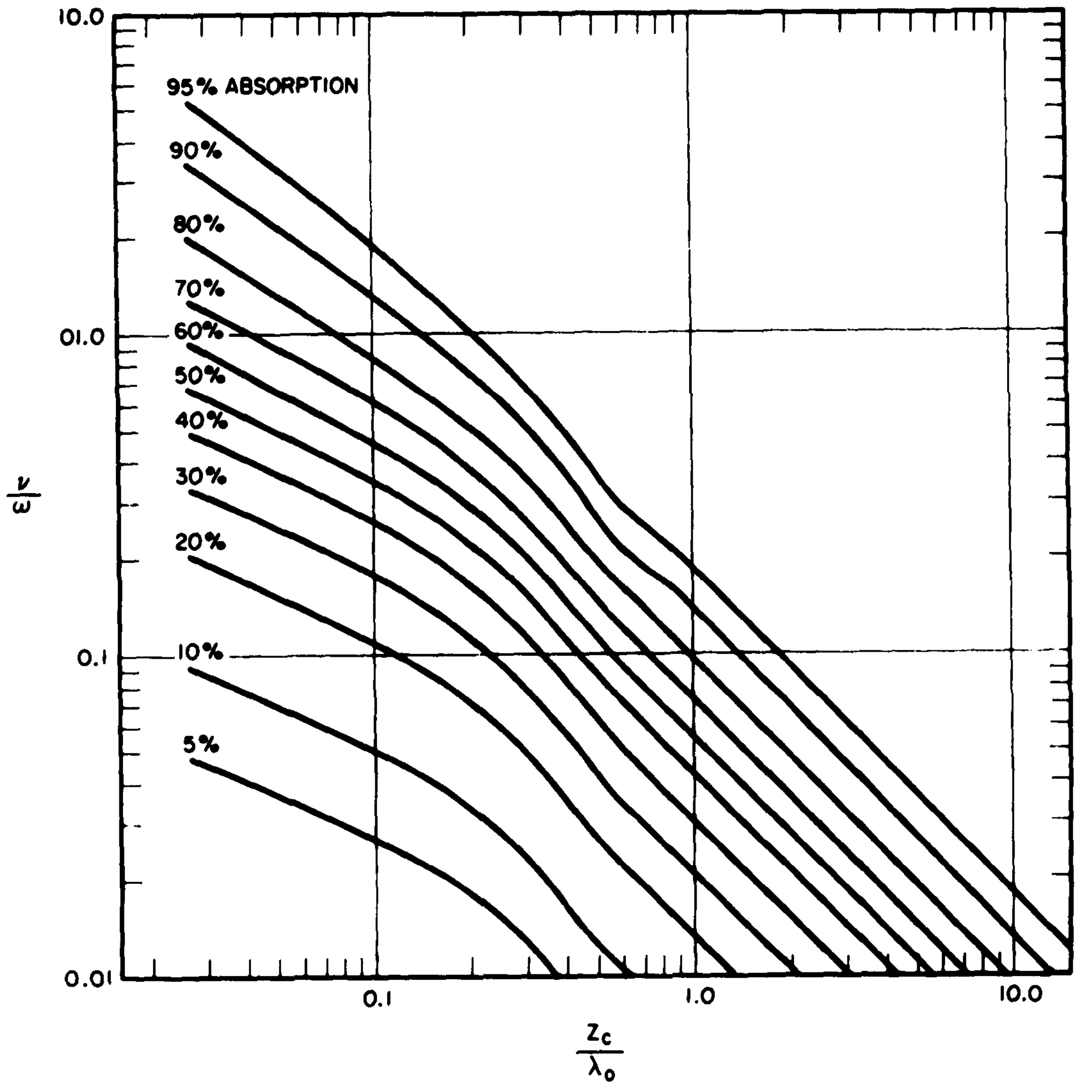

Figure 3 Total absorption in semi-infinite linearly varying plasma (Case I) as function of ν/ω and z_c/λ_o. Total absorption as curve parameter.

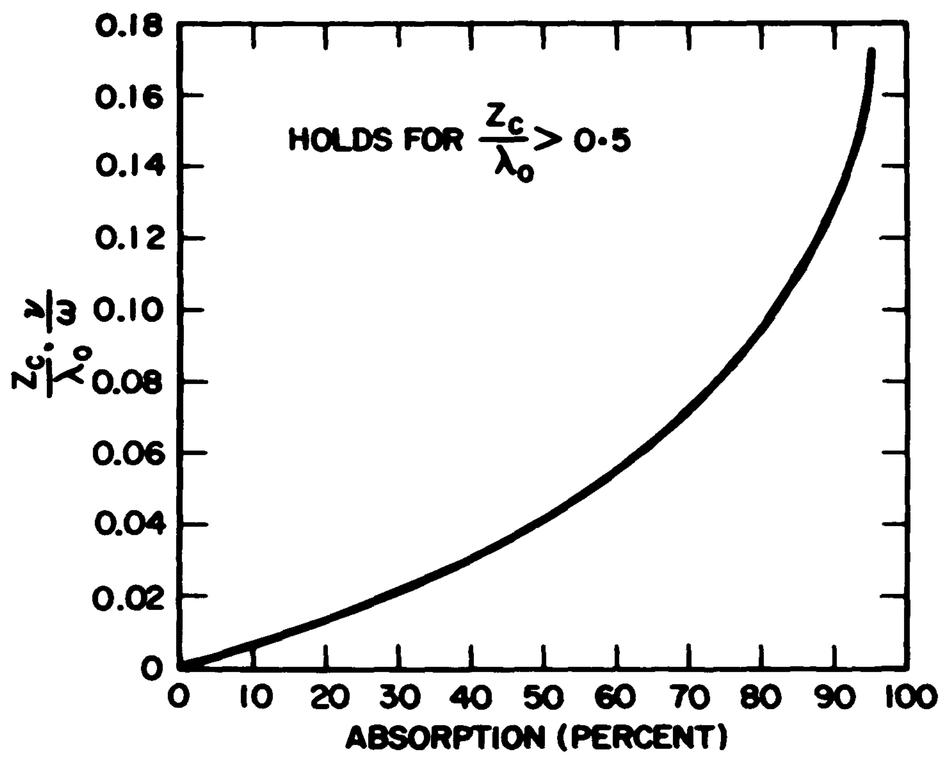

Figure 4 Total absorption in semi-infinite linearly varying plasma (Case II) as function of the product (z_c/λ_o) (ν/ω) for the range $z_c/\lambda_o > 0.5$

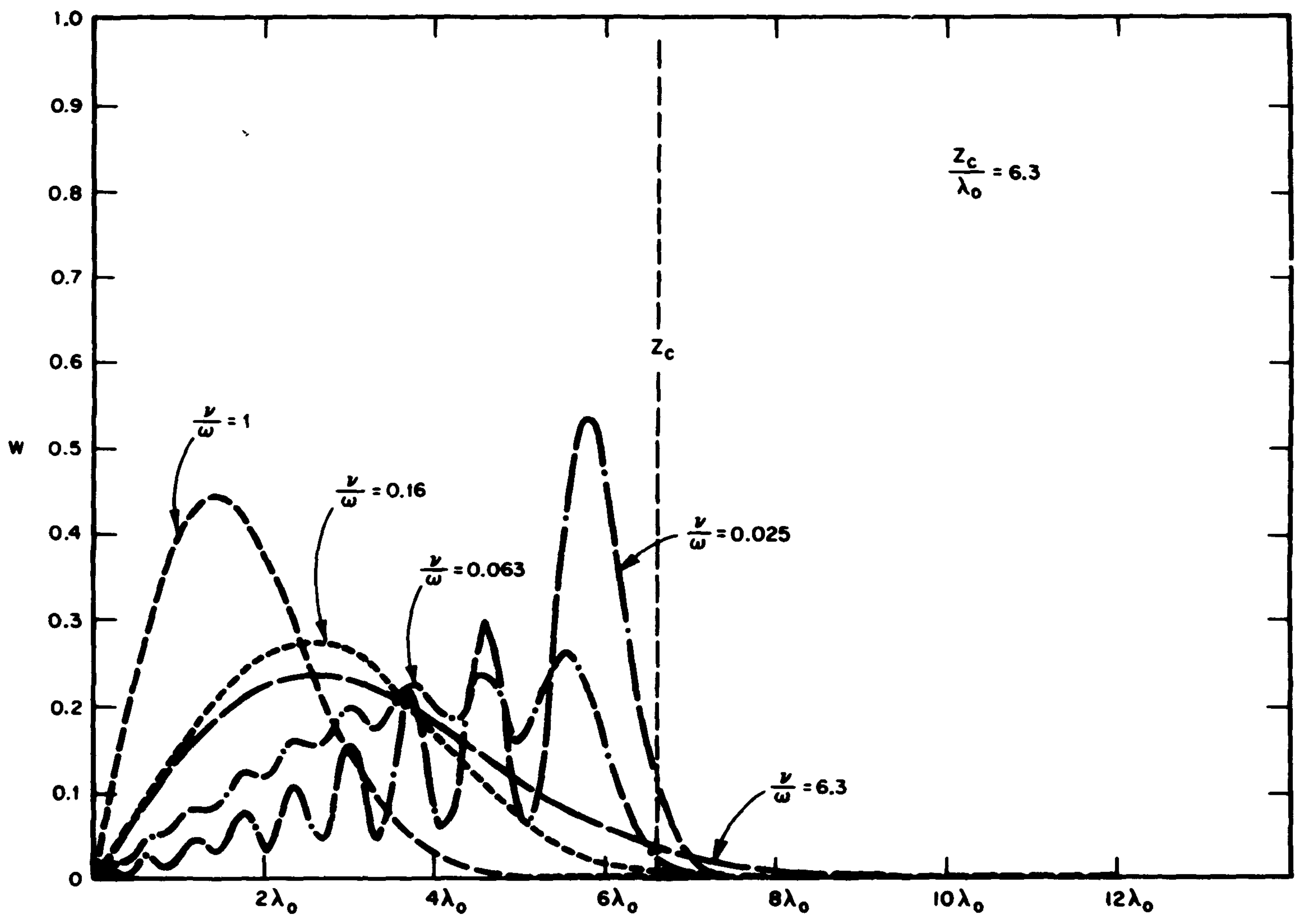

Figure 5a The absorption W as function of the distance z, for semi-infinite linearly varying plasma (Case I)

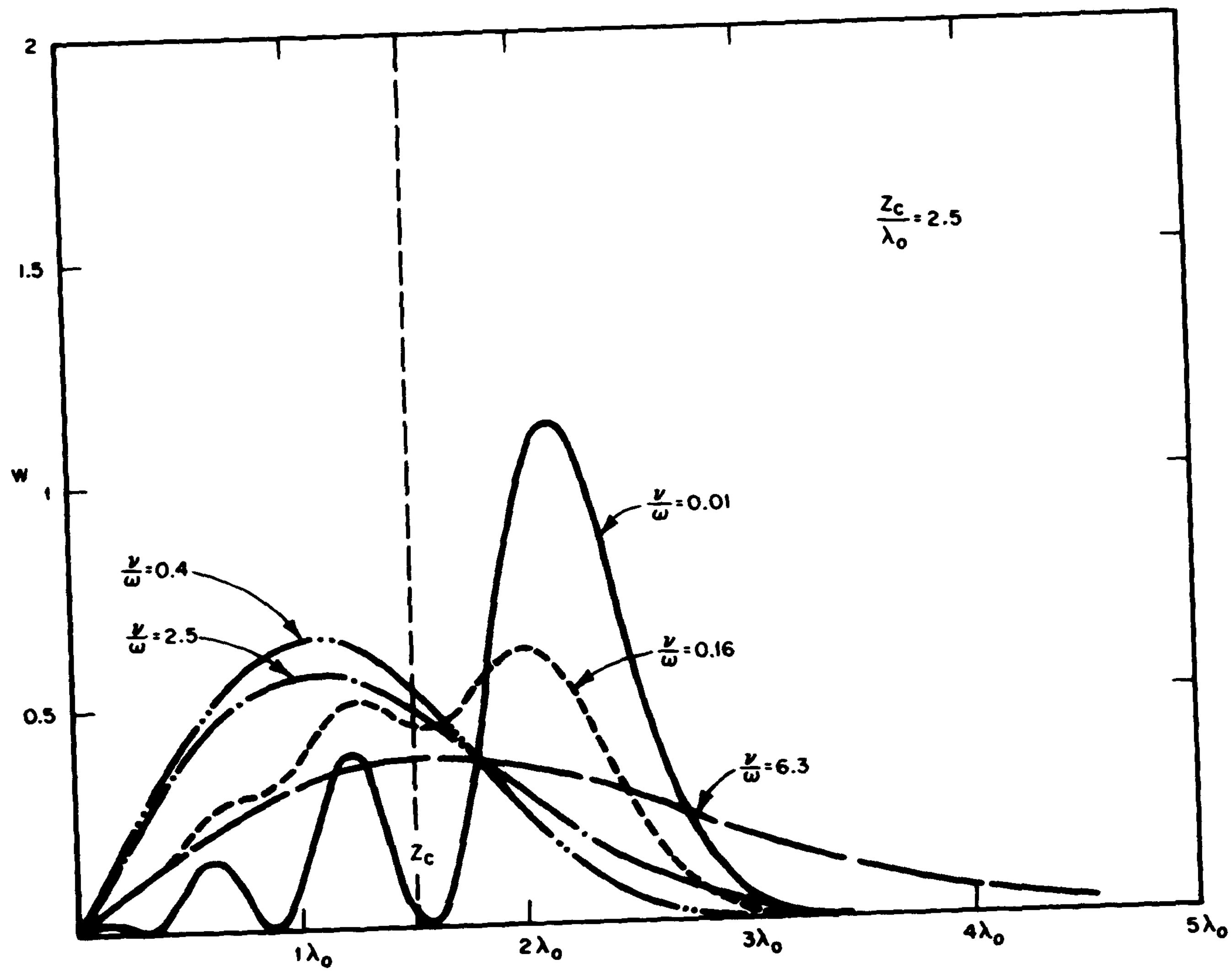

Figure 5b   The absorption W as function of the distance z, for semi-infinite linearly varying plasma (Case I)

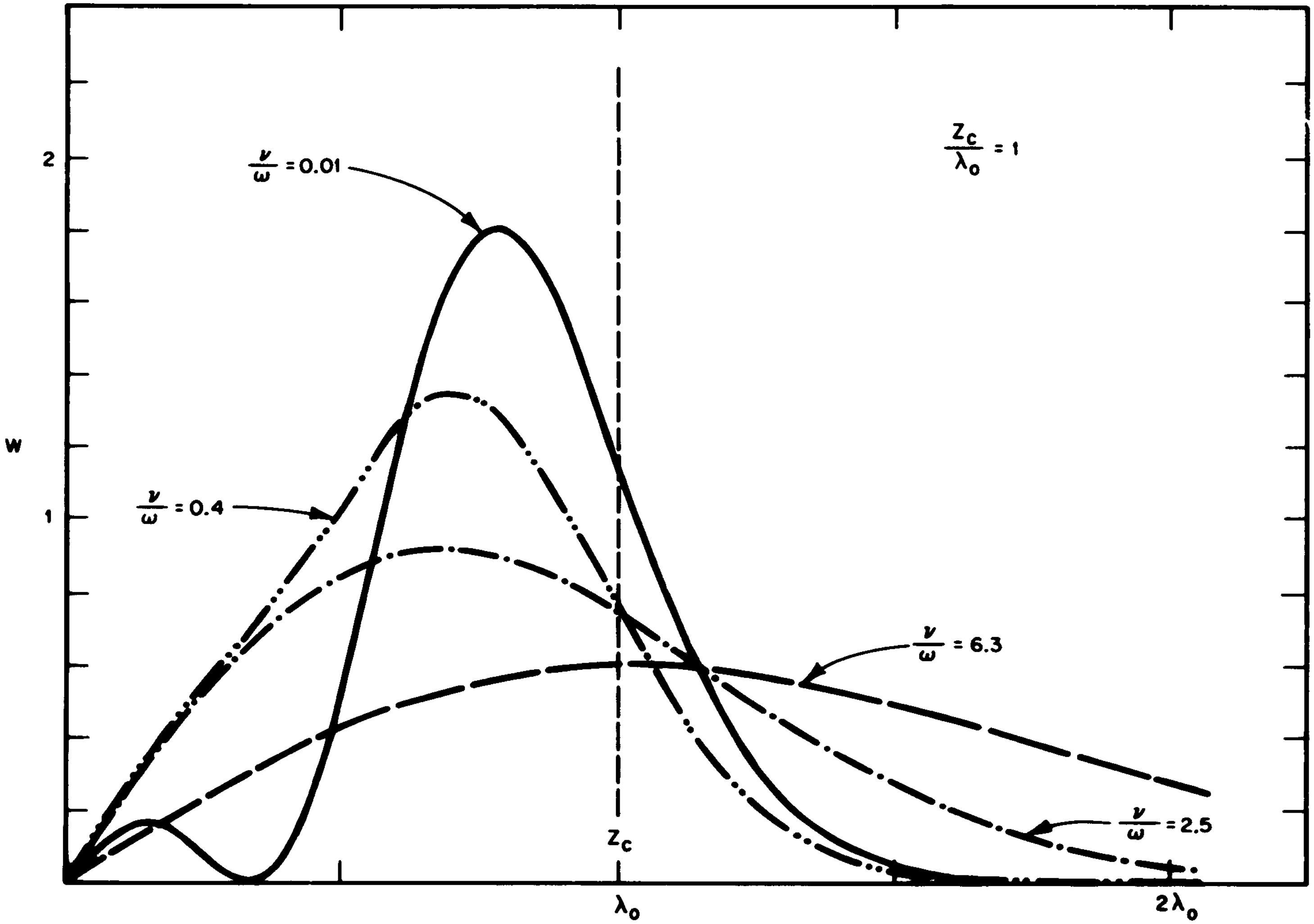

Figure 5c The absorption W as function of the distance z, for semi-infinite linearly varying plasma (Case I)

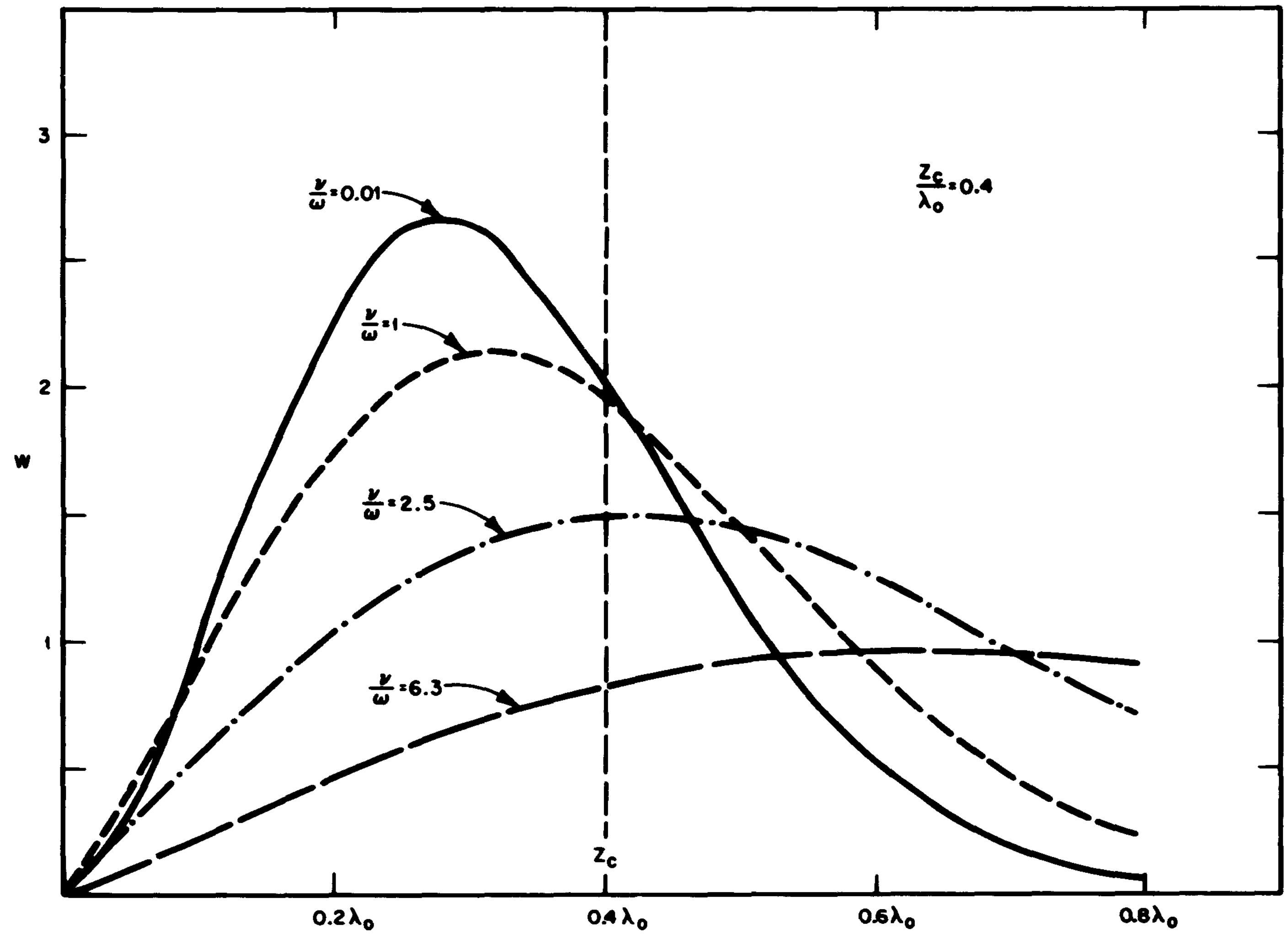

Figure 5d The absorption W as function of the distance z, for semi-infinite linearly varying plasma (Case I)

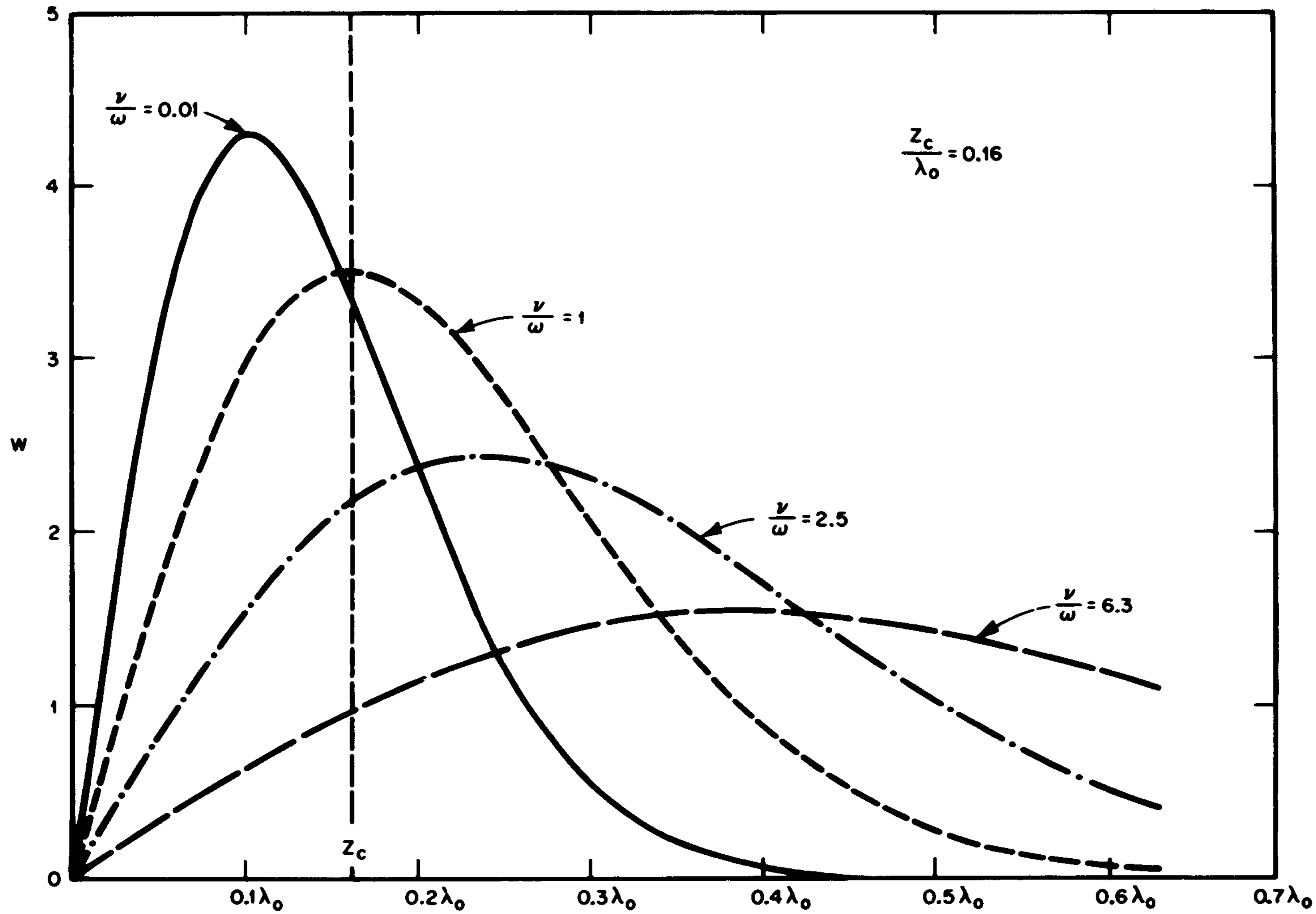

Figure 5e The absorption W as function of the distance z, for semi-infinite linearly varying plasma (Case I)

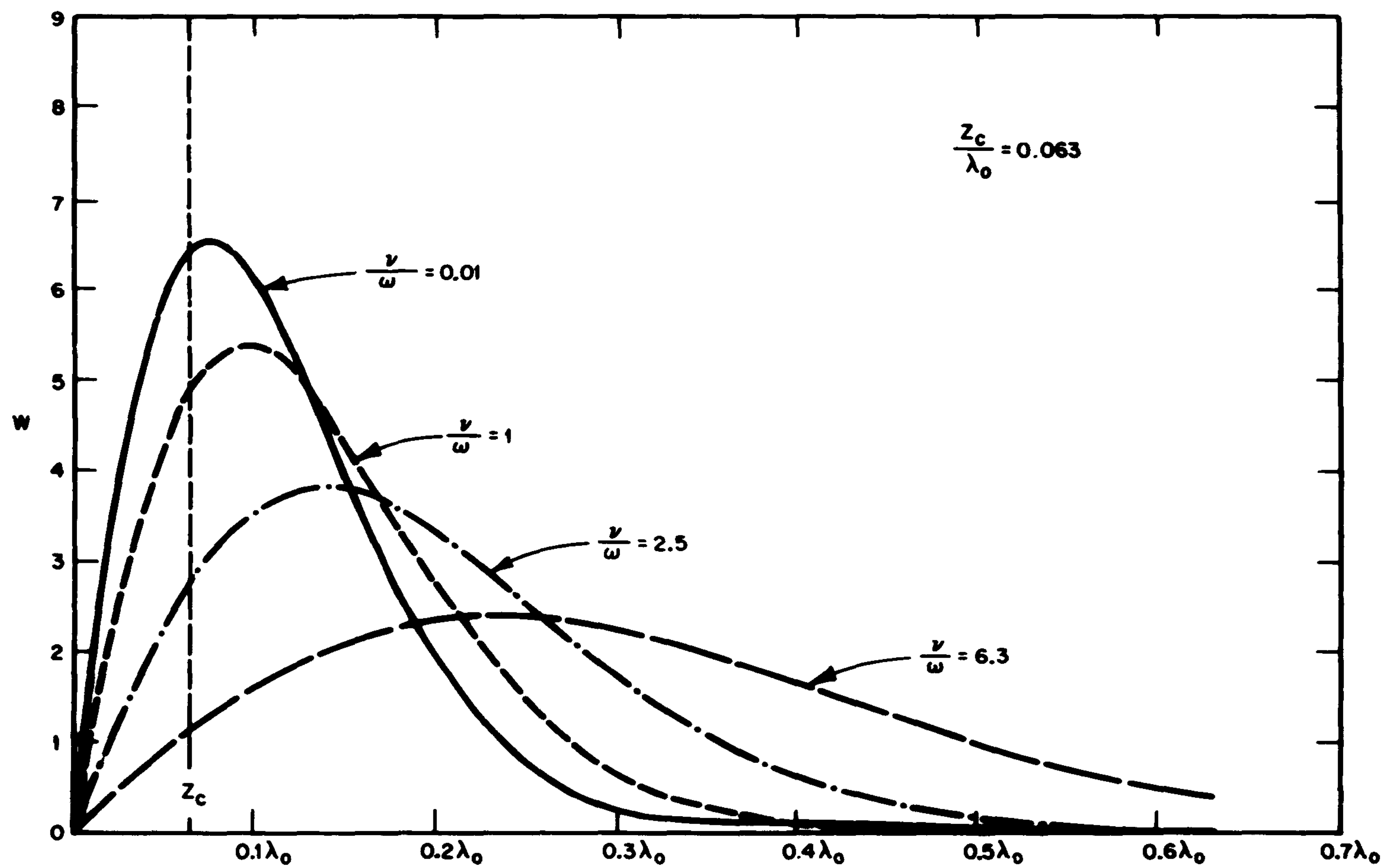

Figure 5f The absorption W as function of the distance z, for semi-infinite linearly varying plasma (Case I)

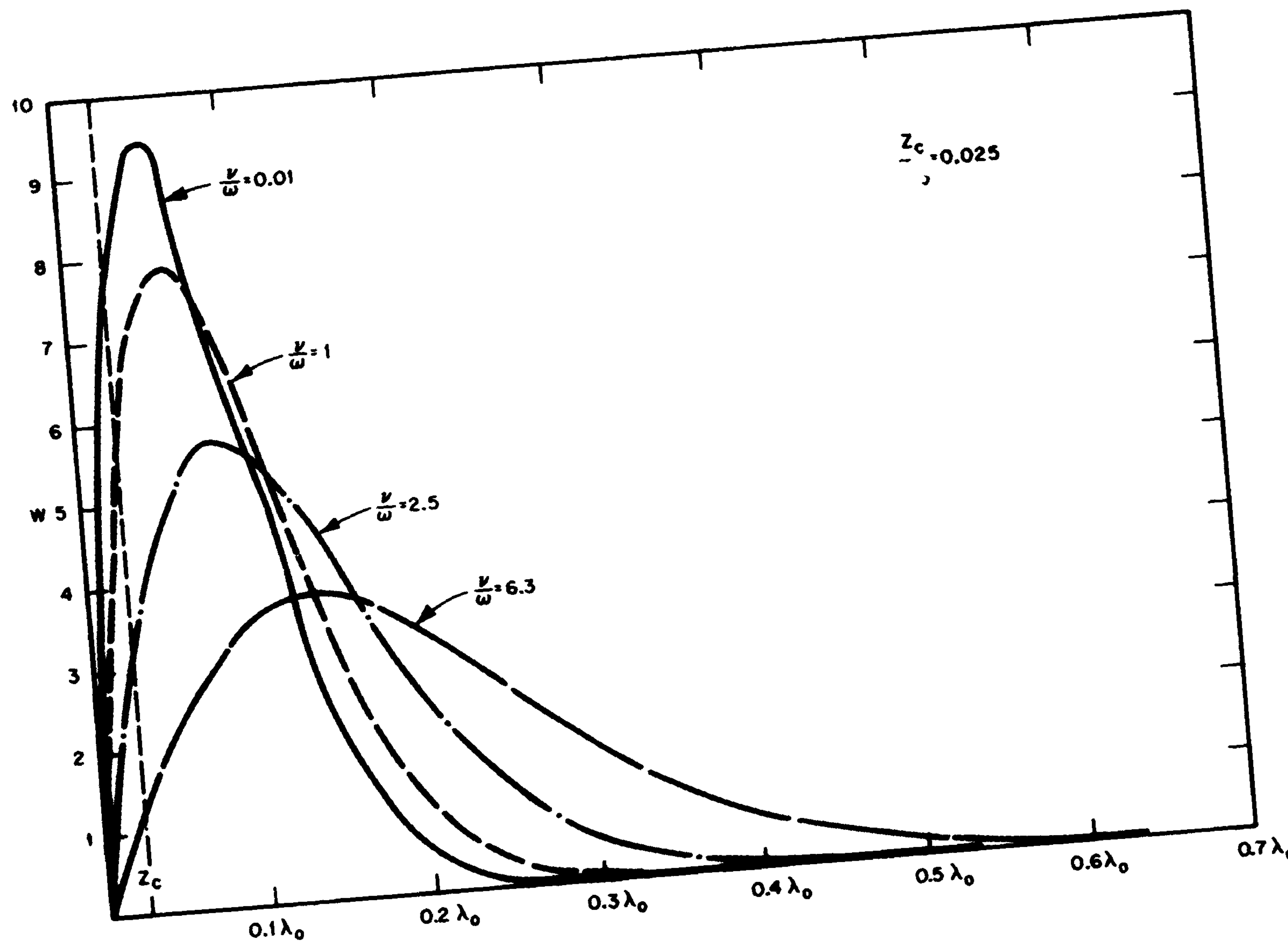

Figure 5g   The absorption W as function of the distance z, for semi-infinite linearly varying plasma (Case I)

CASE II:

Total absorption was computed for various values z_1 of the near boundary for all combinations of the following values:

ν/ω = 0.01, 0.025, 0.063, 0.16, 0.4, 1, 2.5, 6.3

z_c/λ_o = 0.16, 0.4, 1, 2.5, 6.3

z_1/z_c = 0.5, 0.75, 0.9, 1

The results are plotted in Figs. 6a-6e for the Case IIa, and in Figs. 7a-7e for the Case IIb. Each figure corresponds to one particular value of the slope, given in terms of z_c/λ_o, with z_1/z_c as curve parameter. The curve corresponding to $z_1 = 0$, i.e., the curve calculated for the corresponding Case I situation, is added for comparison. An inspection of the curves thus gives an immediate indication of the effect of a shift in the near boundary position as given by z_1. Curves have been omitted wherever they do not differ appreciably from the $z_1 = 0$ curves, or to avoid overcrowding.

CASE III:

Total absorption was computed for various values z_2 of the far boundary for all combinations of the following values:

ν/ω = 0.01, 0.025, 0.063, 0.16, 0.4, 1, 2.5, 6.3

z_c/λ_o = 0.16, 0.4, 1, 2.5, 6.3, 16

z_2/z_c = 1, 1.1, 1.25, 1.5, 2

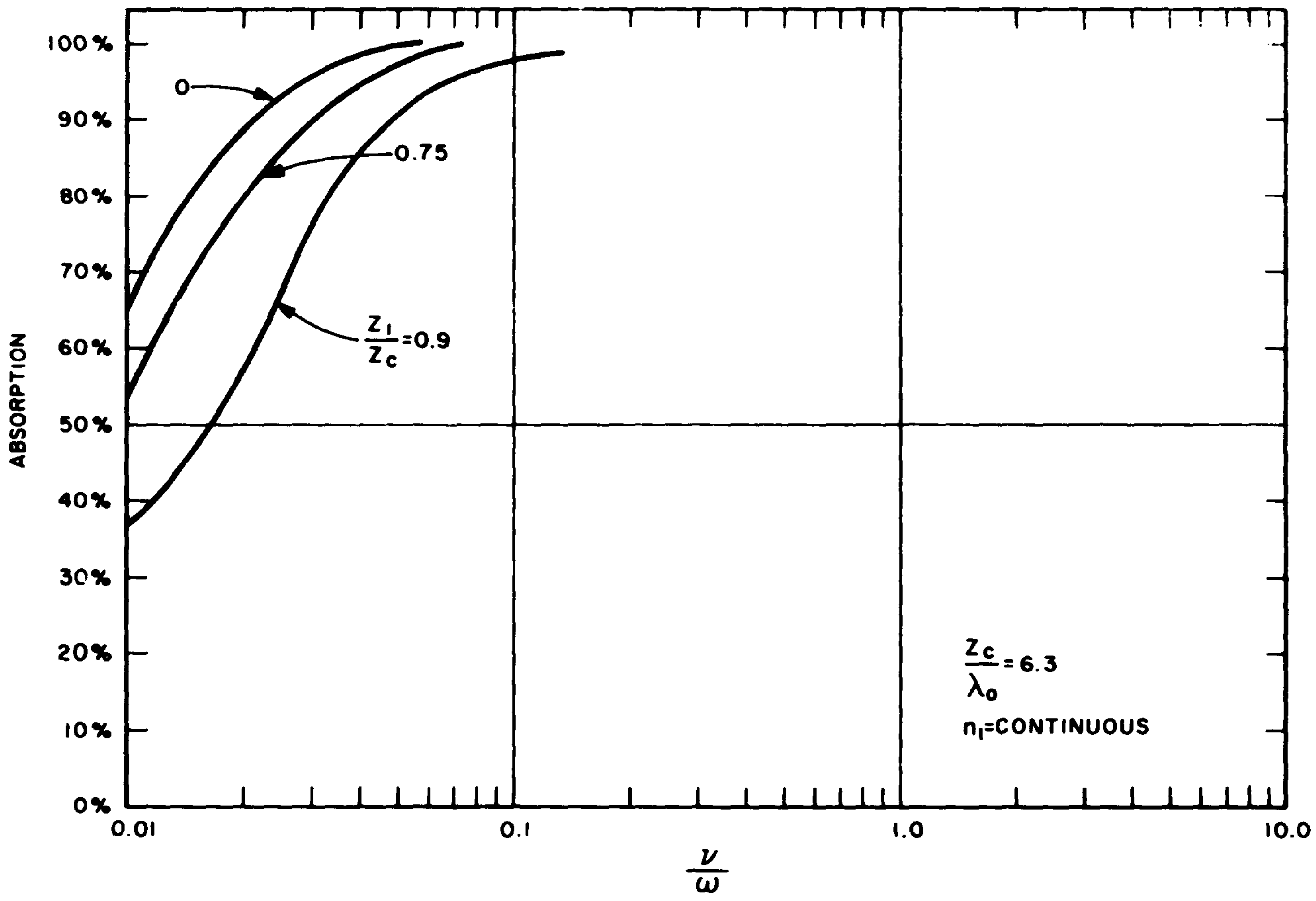

Figure 6a Total absorption for the Case IIa

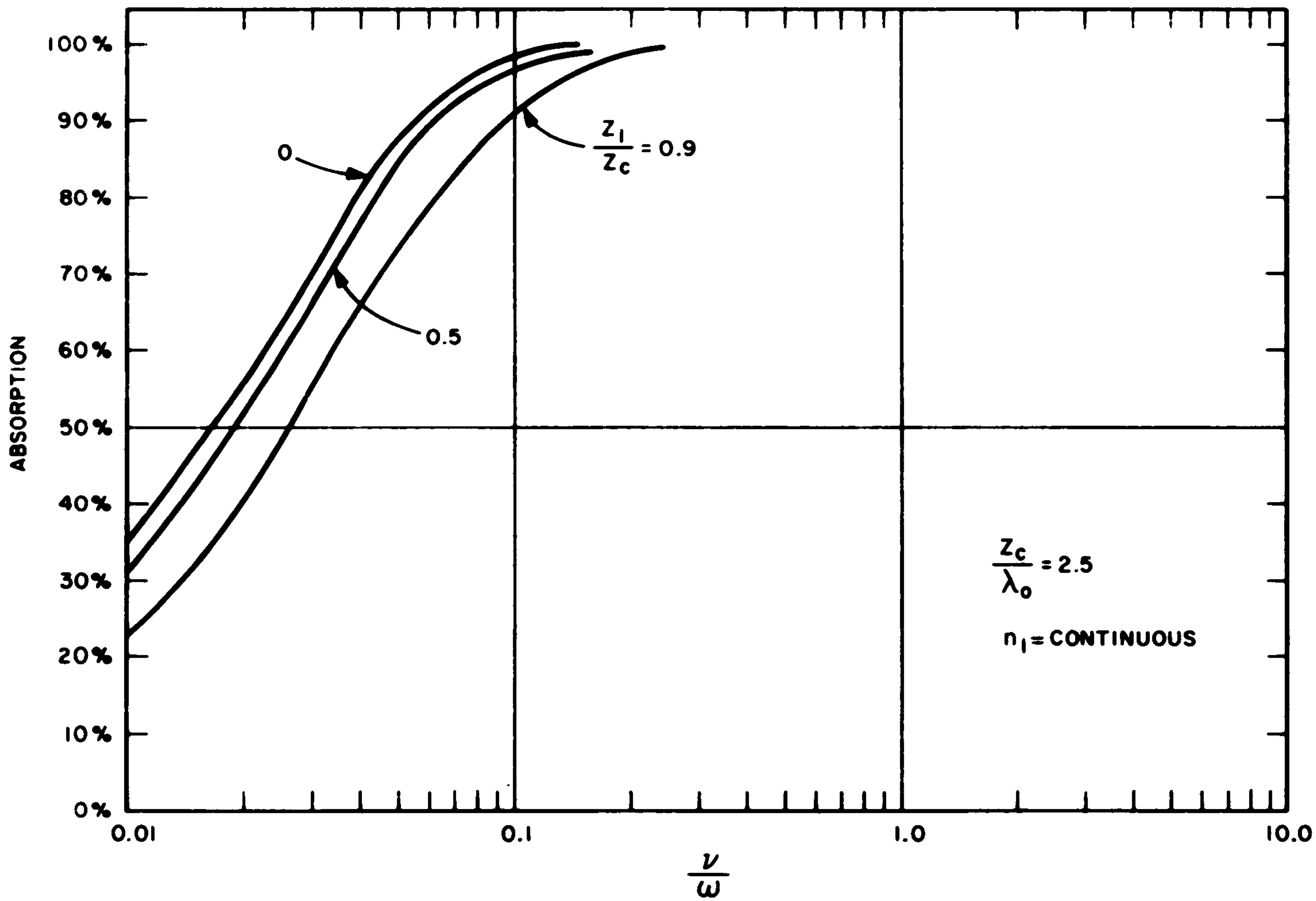

Figure 6b Total absorption for the Case IIa

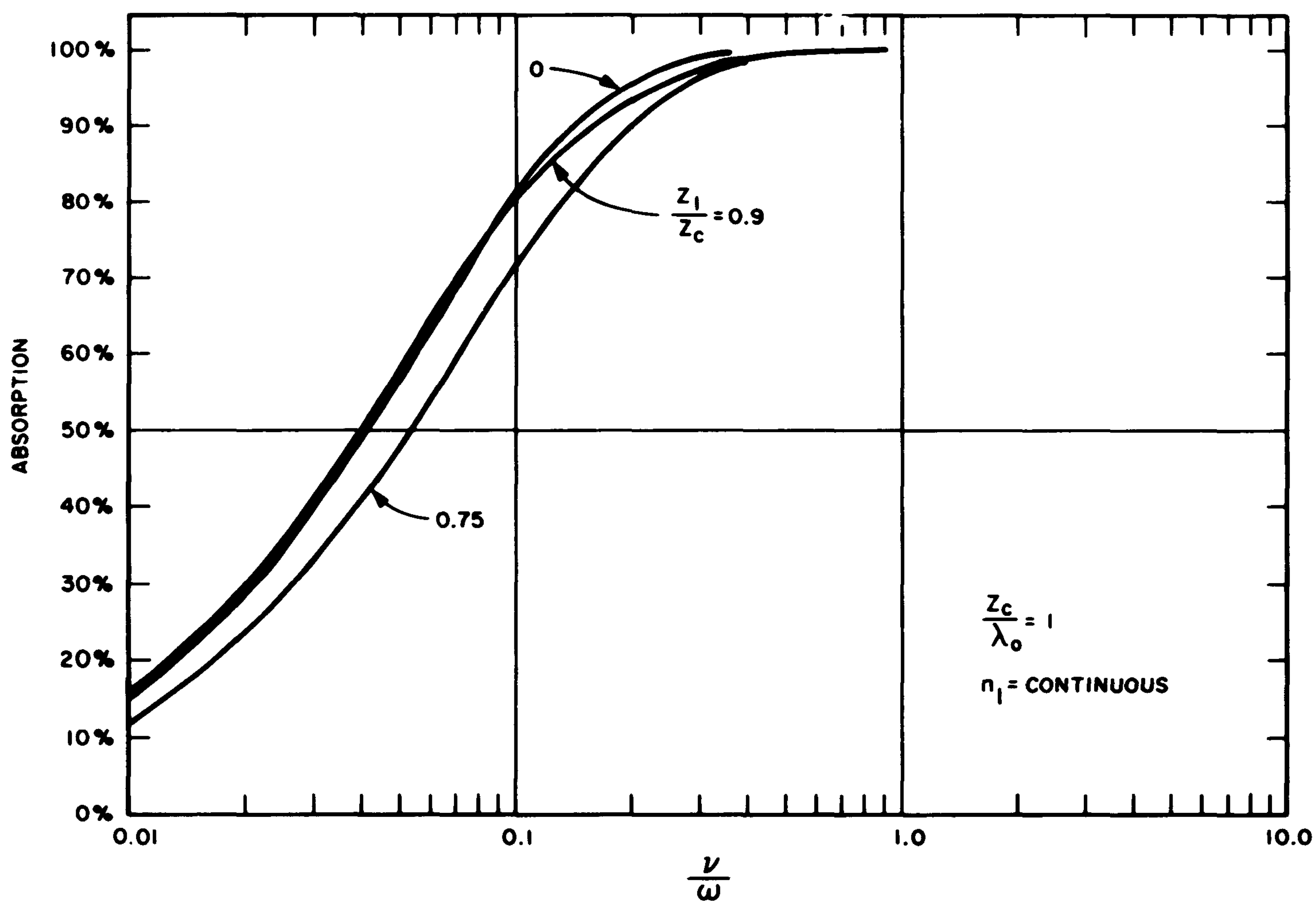

Figure 6c Total absorption for the Case IIa

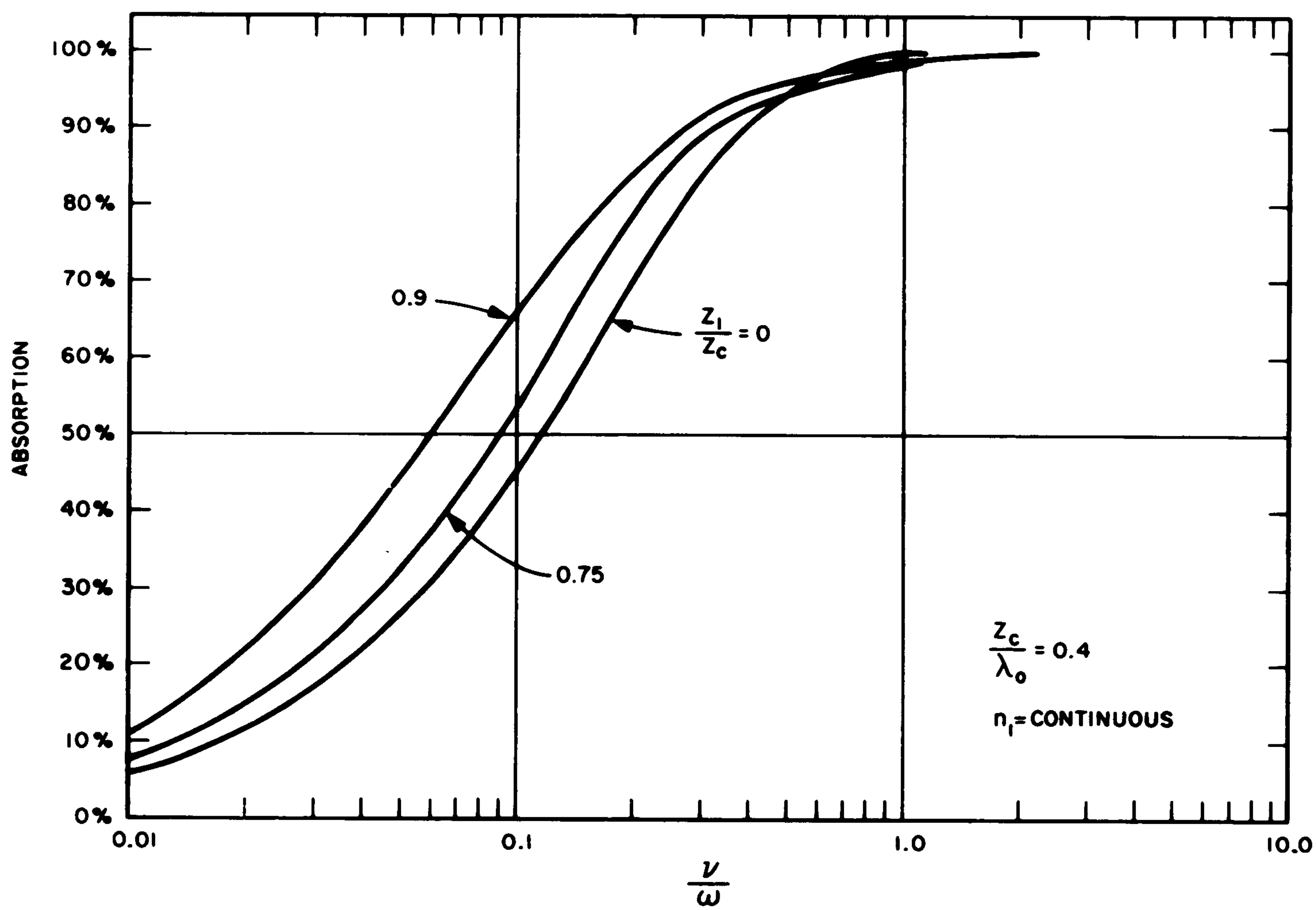

Figure 6d Total absorption for the Case IIa

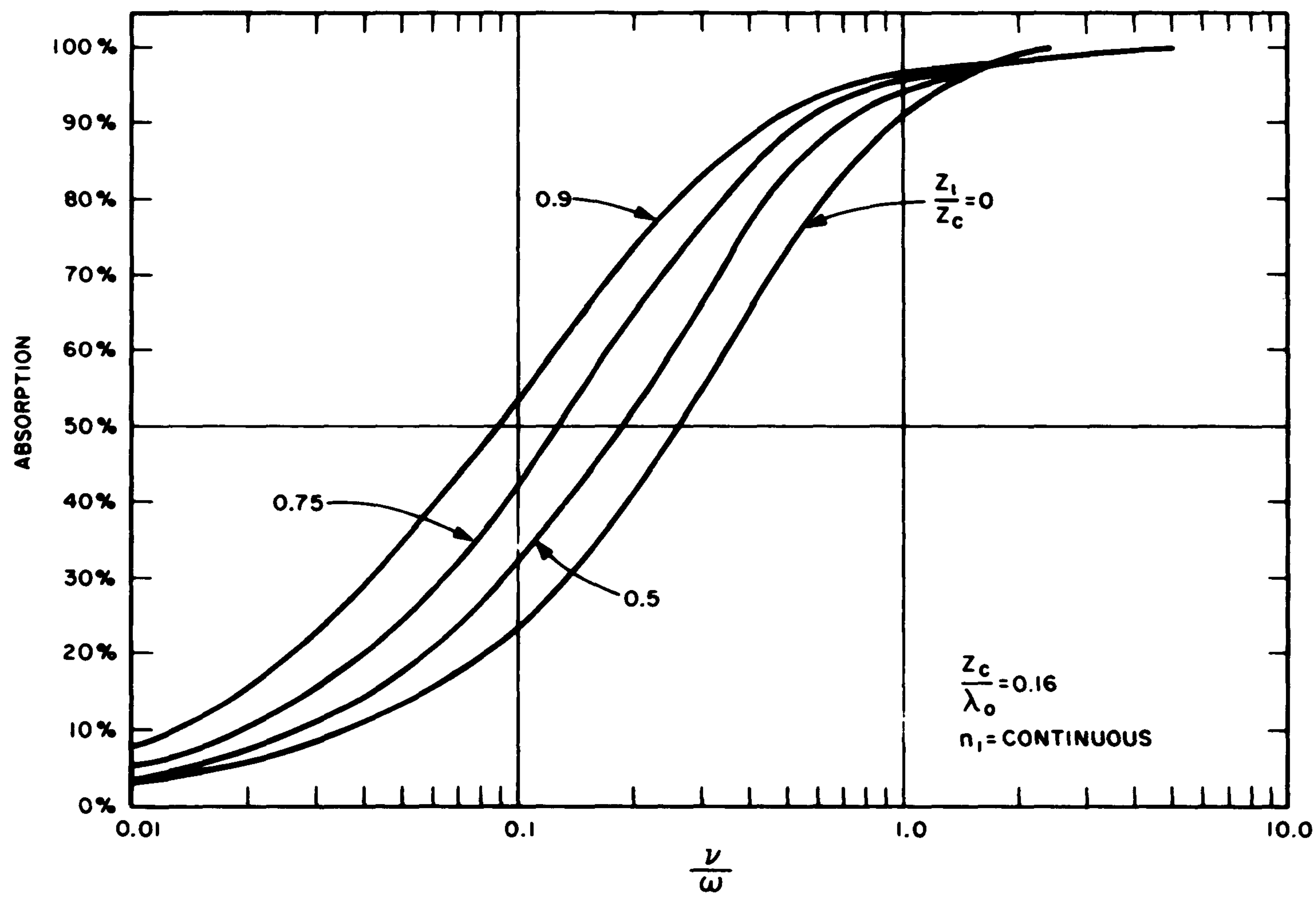

Figure 6e Total absorption for the Case IIa

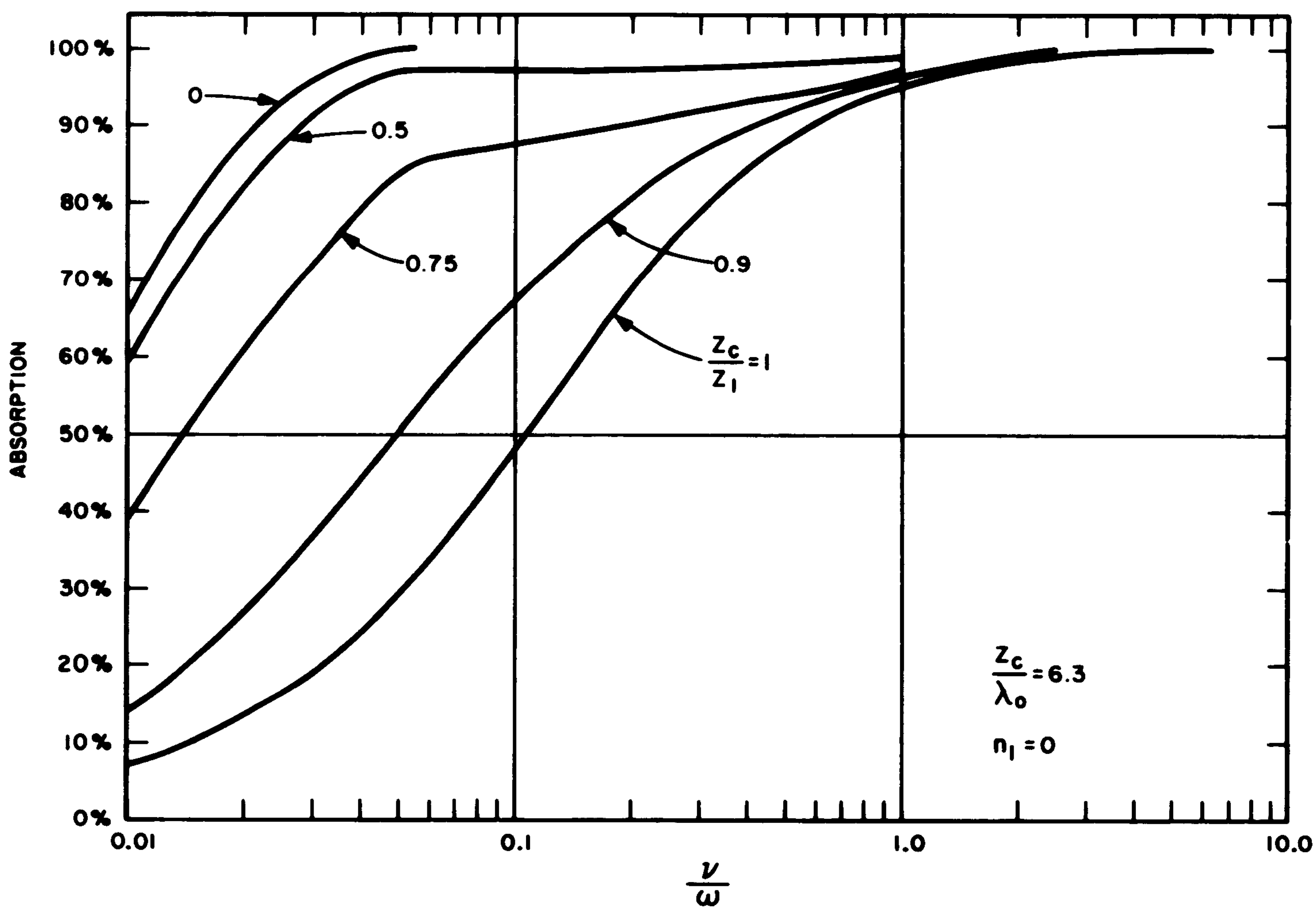

Figure 7a Total absorption for the Case IIb

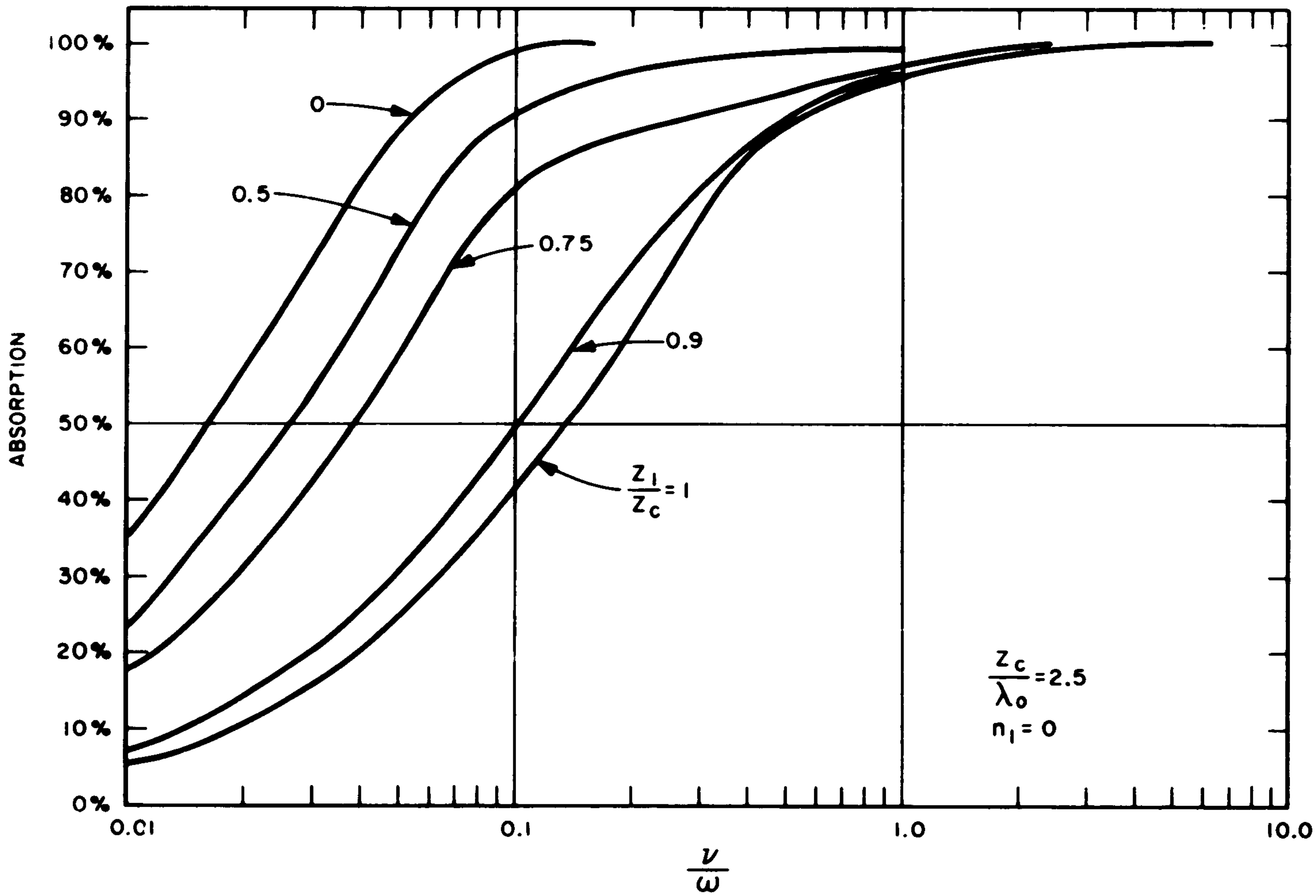

Figure 7b Total absorption for the Case IIb

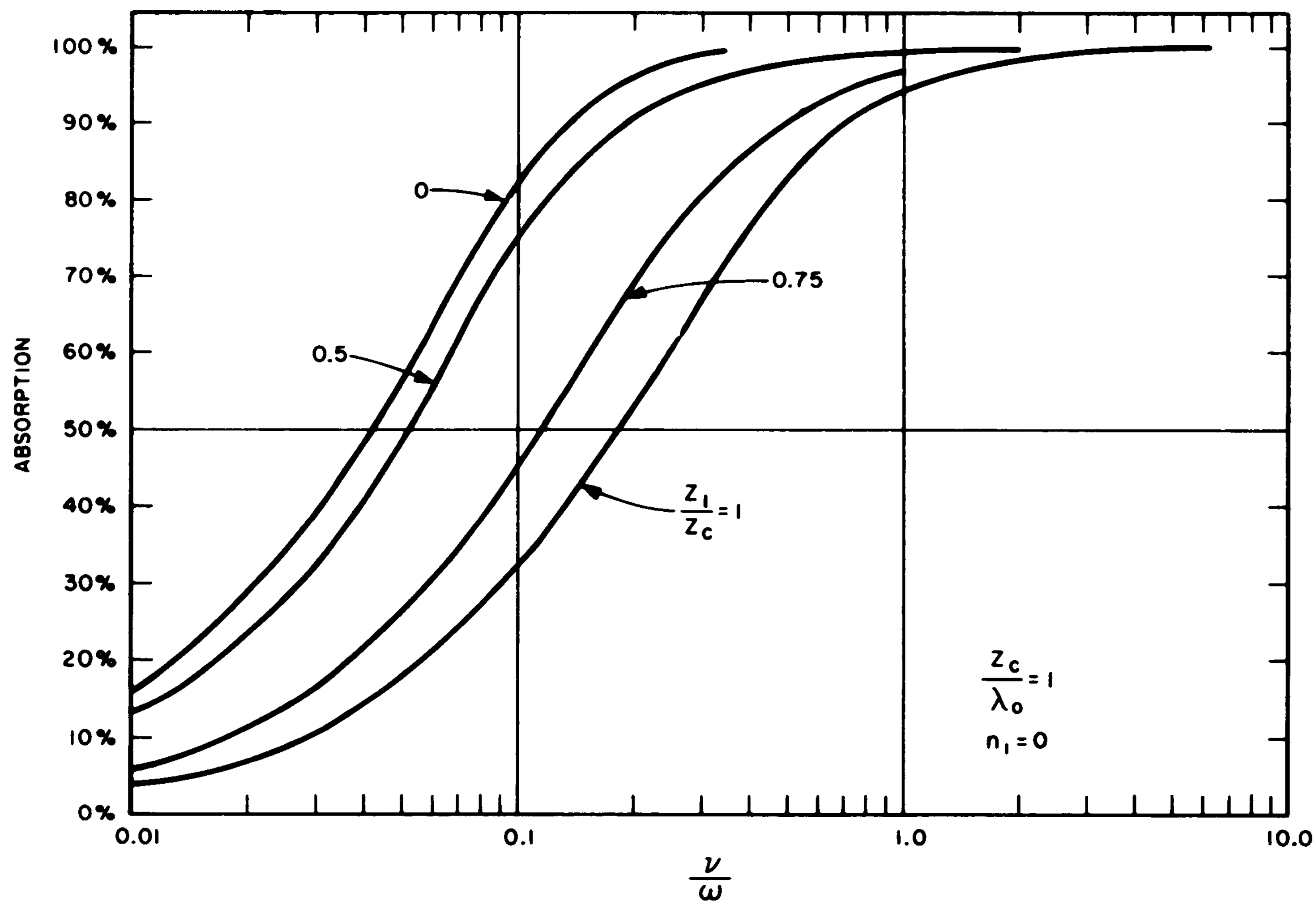

Figure 7c Total absorption for the Case IIb

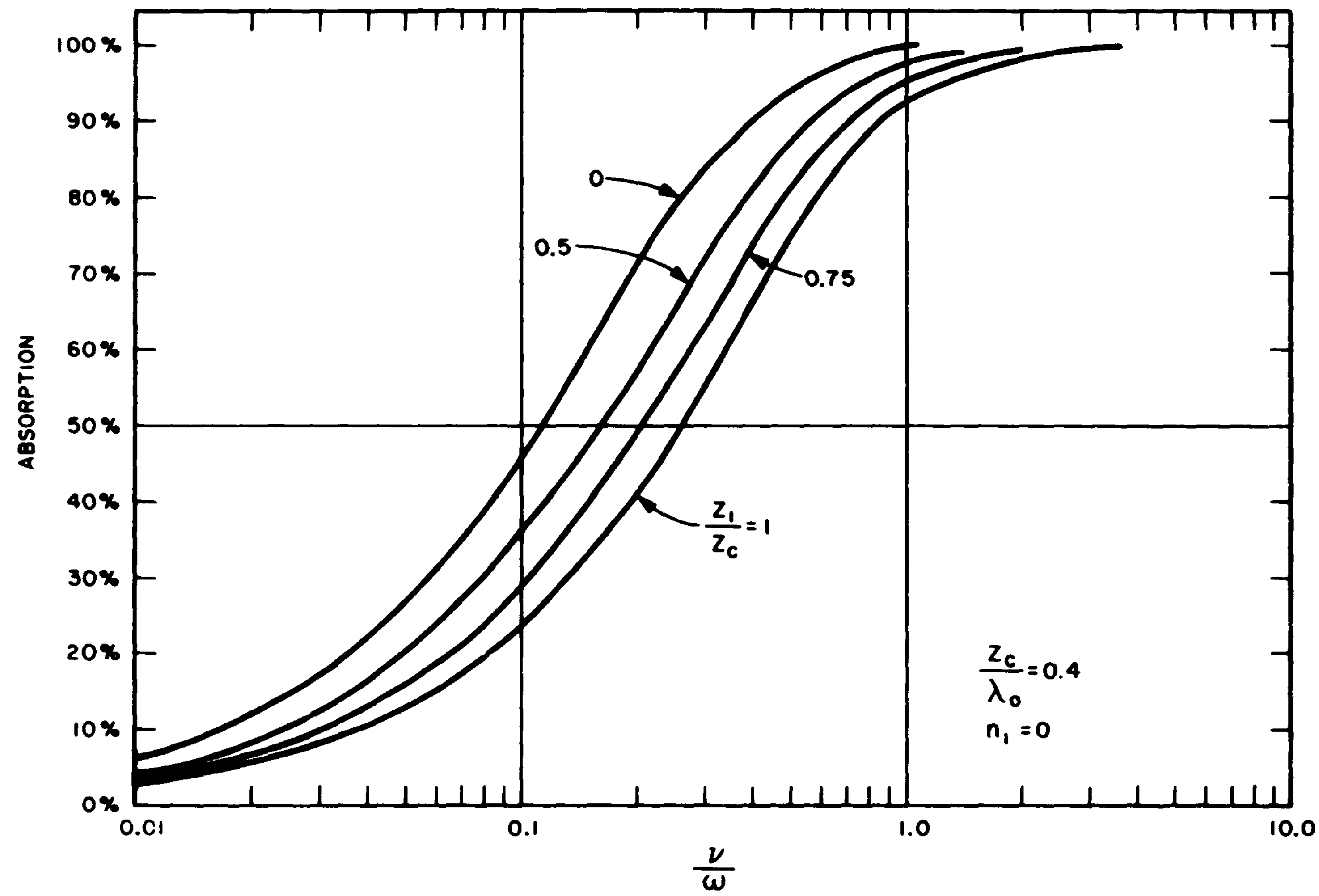

Figure 7d **Total absorption for the Case IIb**

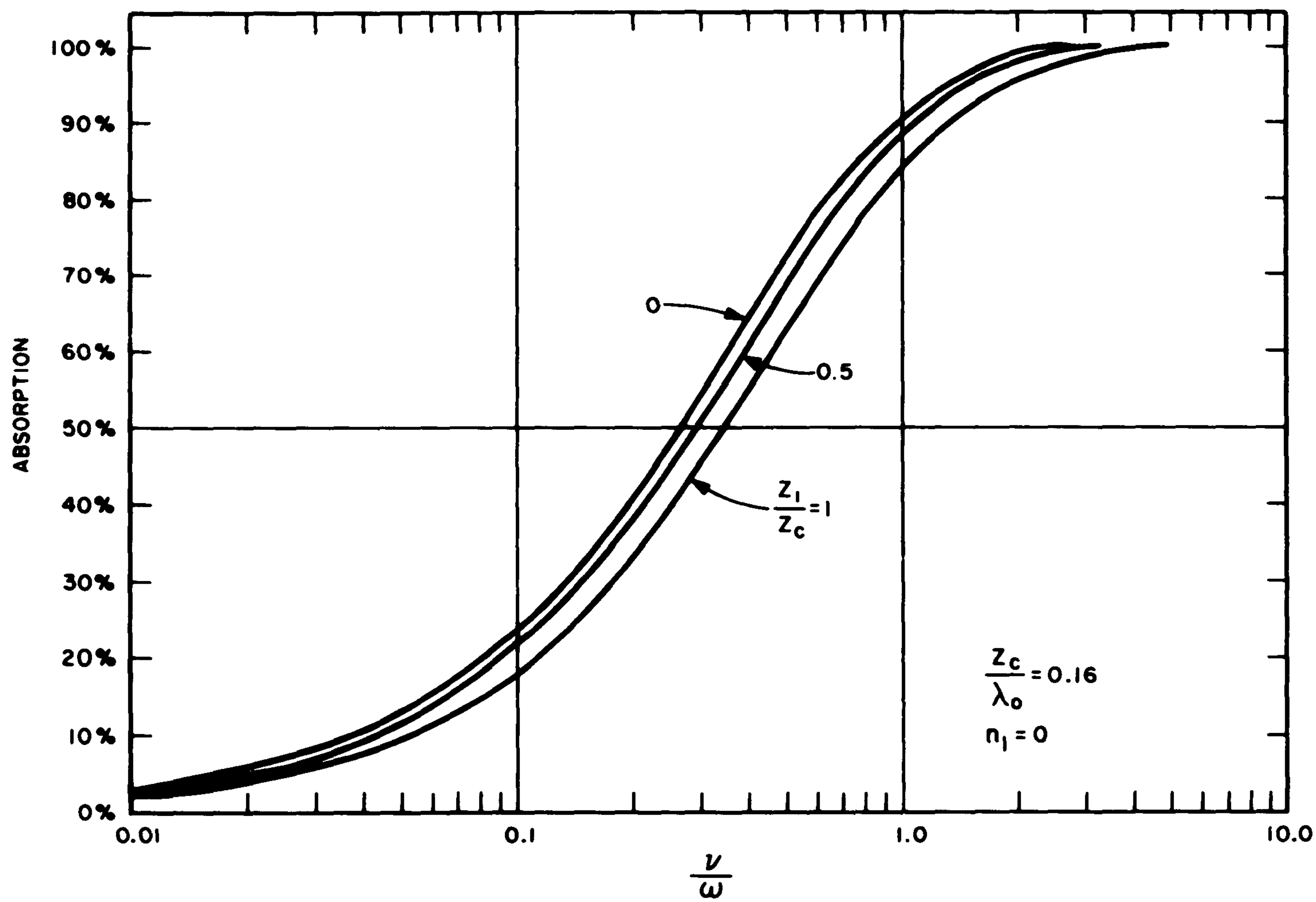

Figure 7e Total absorption for the Case IIb

The results are plotted in Figs. 8a-8f for the Case IIa, in Figs. 9a-9f for the Case IIb and in Figs. 10a-10f for the Case IIIc. Each figure corresponds to a particular value of the slope, given in terms of z_c/λ_o, with z_2/z_c as curve parameter. The curve correspondong to $z_2 = \infty$, i.e., the curve calculated for the corresponding Case I situation, is added for comparison. An inspection of the curves thus gives immediate indication of the effect of a shift in the far boundary coordinate, z_2. Curves have been omitted wherever they do not differ appreciably from the $z_2 = \infty$ curves, or to avoid overcrowding.

IV. <u>DISCUSSION OF DATA</u>

<u>CASE I:</u>

Fig. 2 displays the total absorption for Case I as a function of ν/ω and z_c/λ_o. The range of parameters is a very wide one, ν/ω varying from 0.01 to 6.3 and z_c/λ_o from 0.025 to 16. The values at the extreme ends are of little practical interest but are included in order to complete the picture. Very high values of z_c/λ_o correspond to very slow rate of increase of the density n, i.e., to a regime where the WKB approximation can be applied, and where absorption is practically complete. Very low values of z_c/λ_o correspond to a very steep gradient, a condition approaching a discontinuous abrupt rise. The value of the present data is in providing information in the range where absorption is substantial but not complete, i.e., the region of intermediate values of z_c/λ_o. The data of Fig. 2 is transcribed in Fig. 3 by

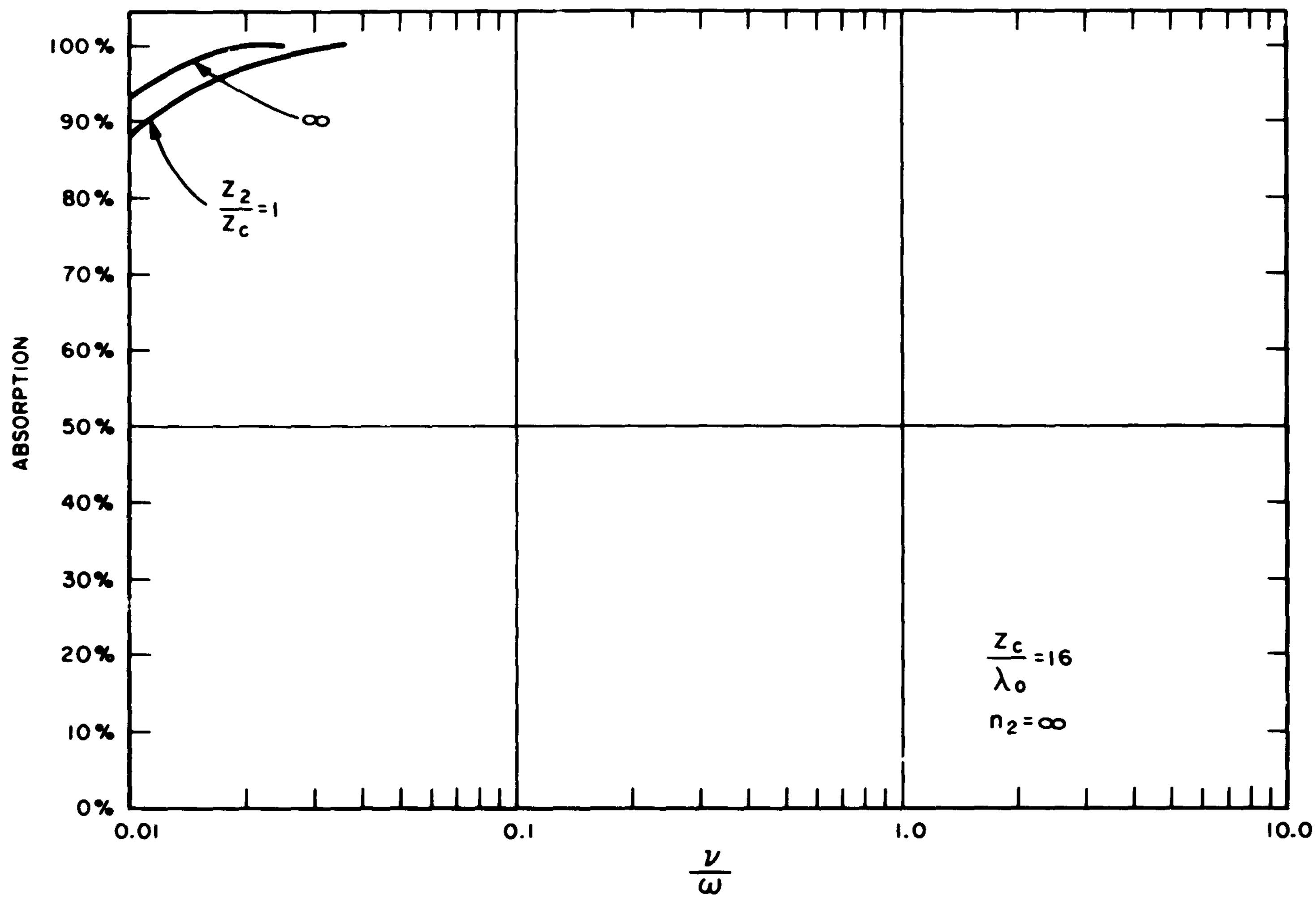

Figure 8a Total absorption for the Case IIIa

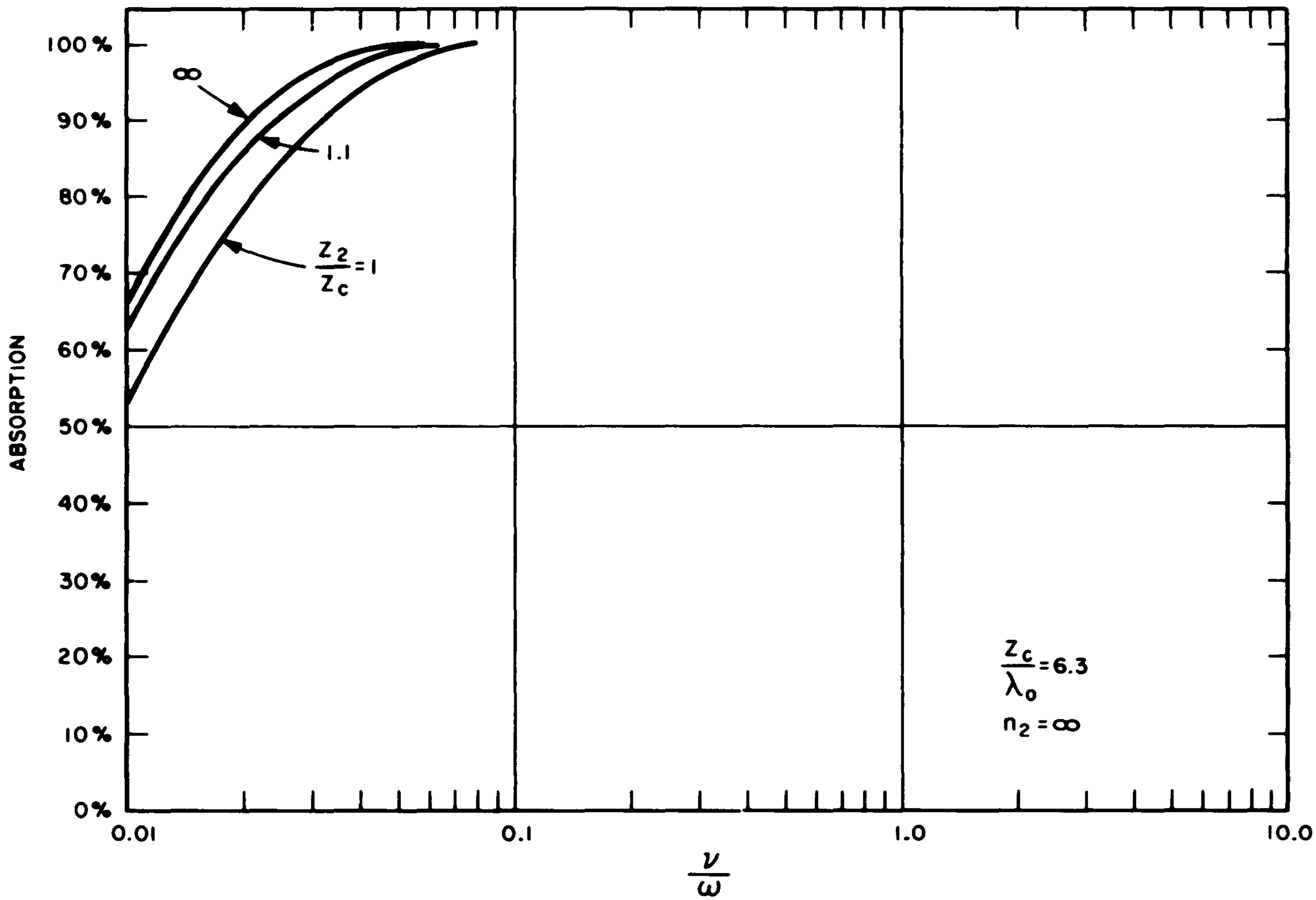

Figure 8b Total absorption for the Case IIIa

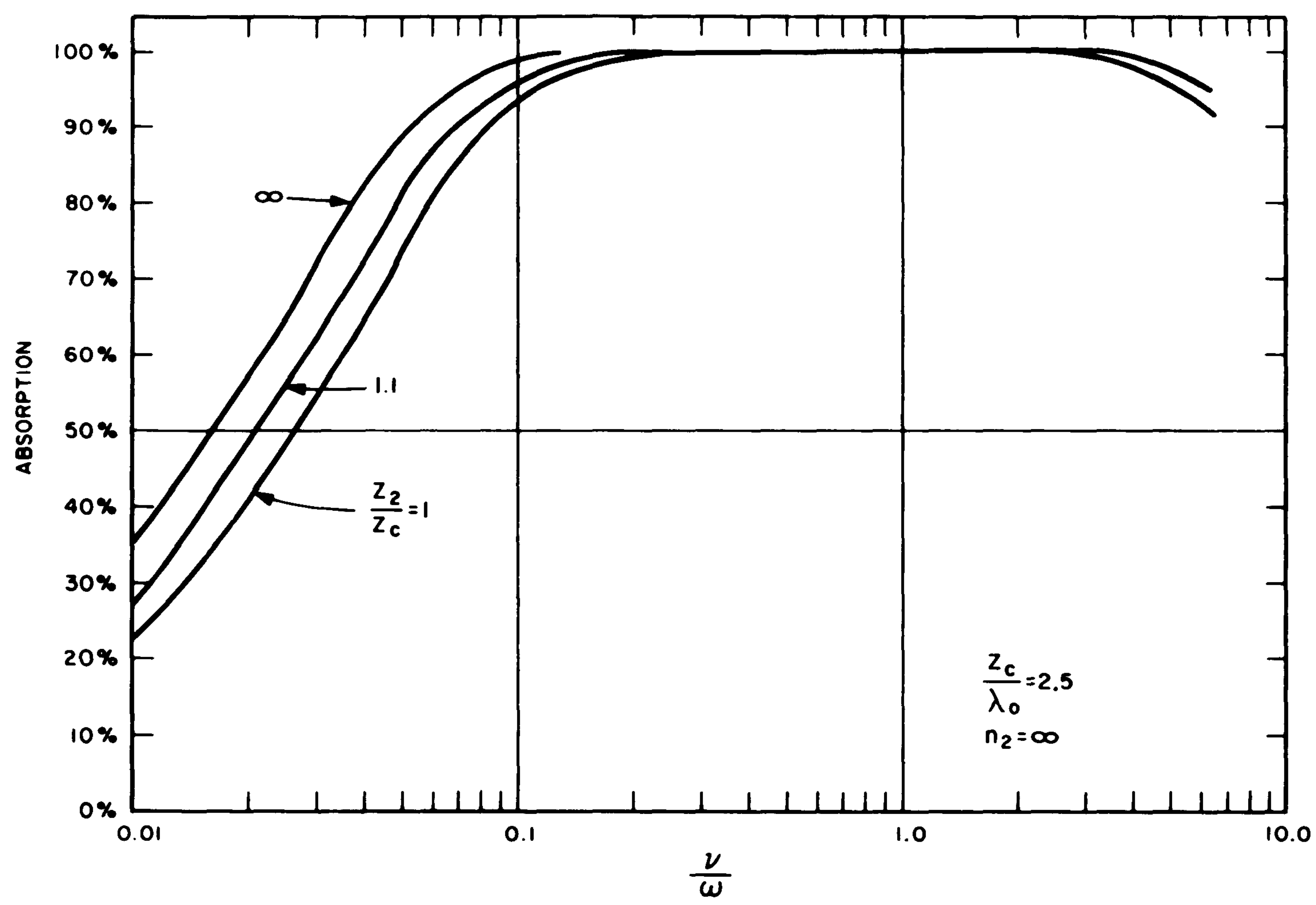

Figure 8c Total absorption for the Case IIIa

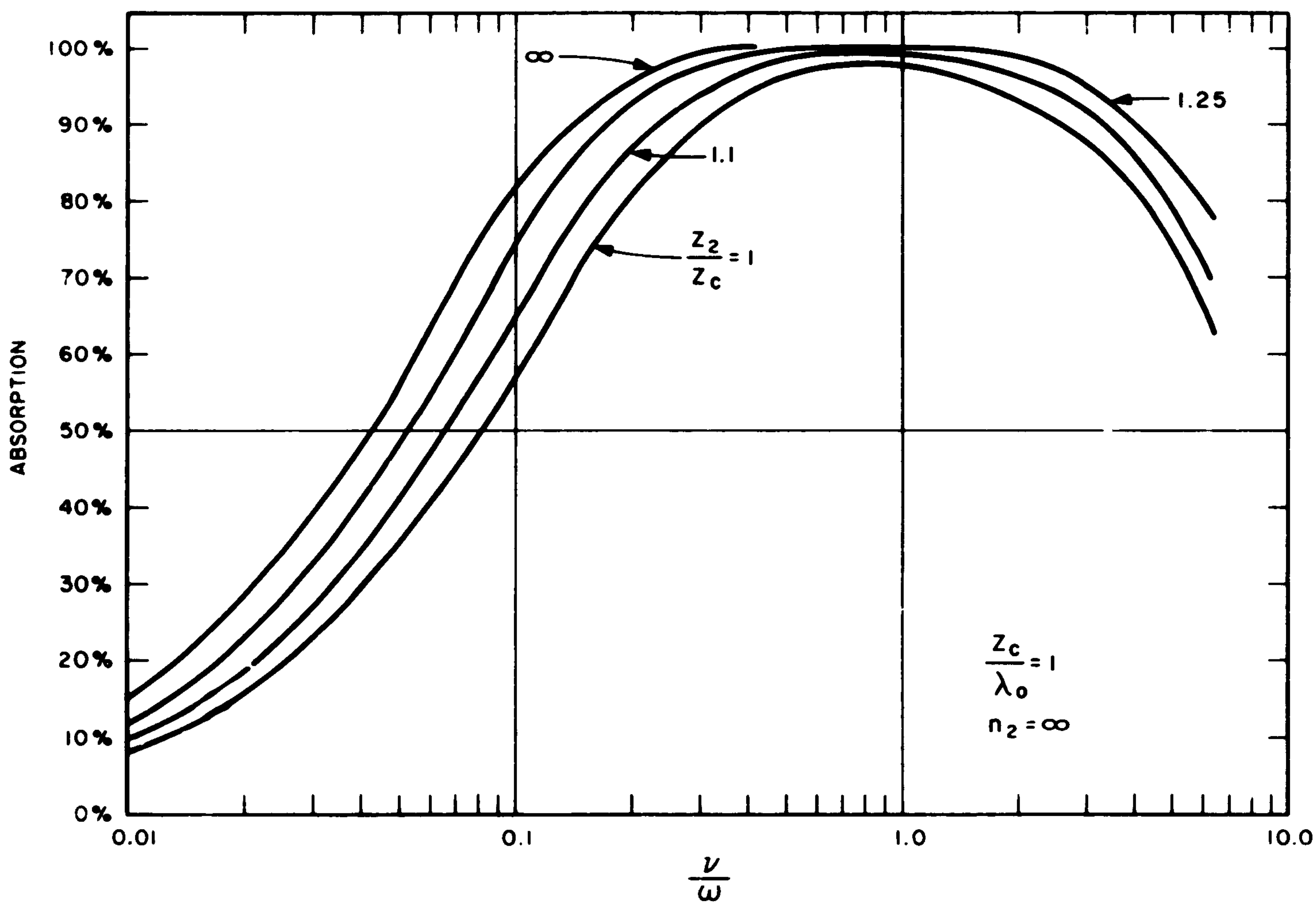

Figure 8d Total absorption for the Case IIIa

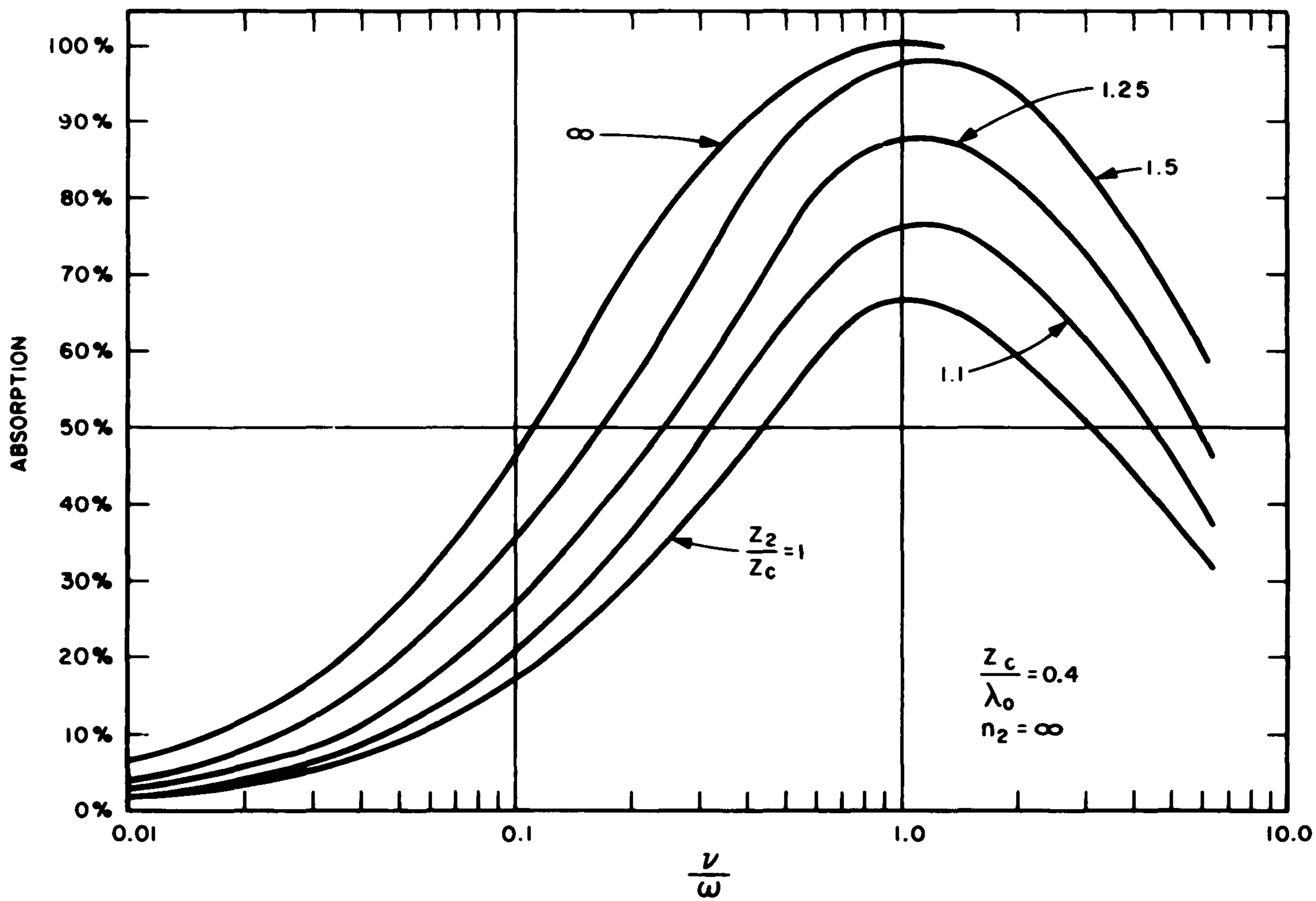

Figure 8e   Total absorption for the Case IIIa

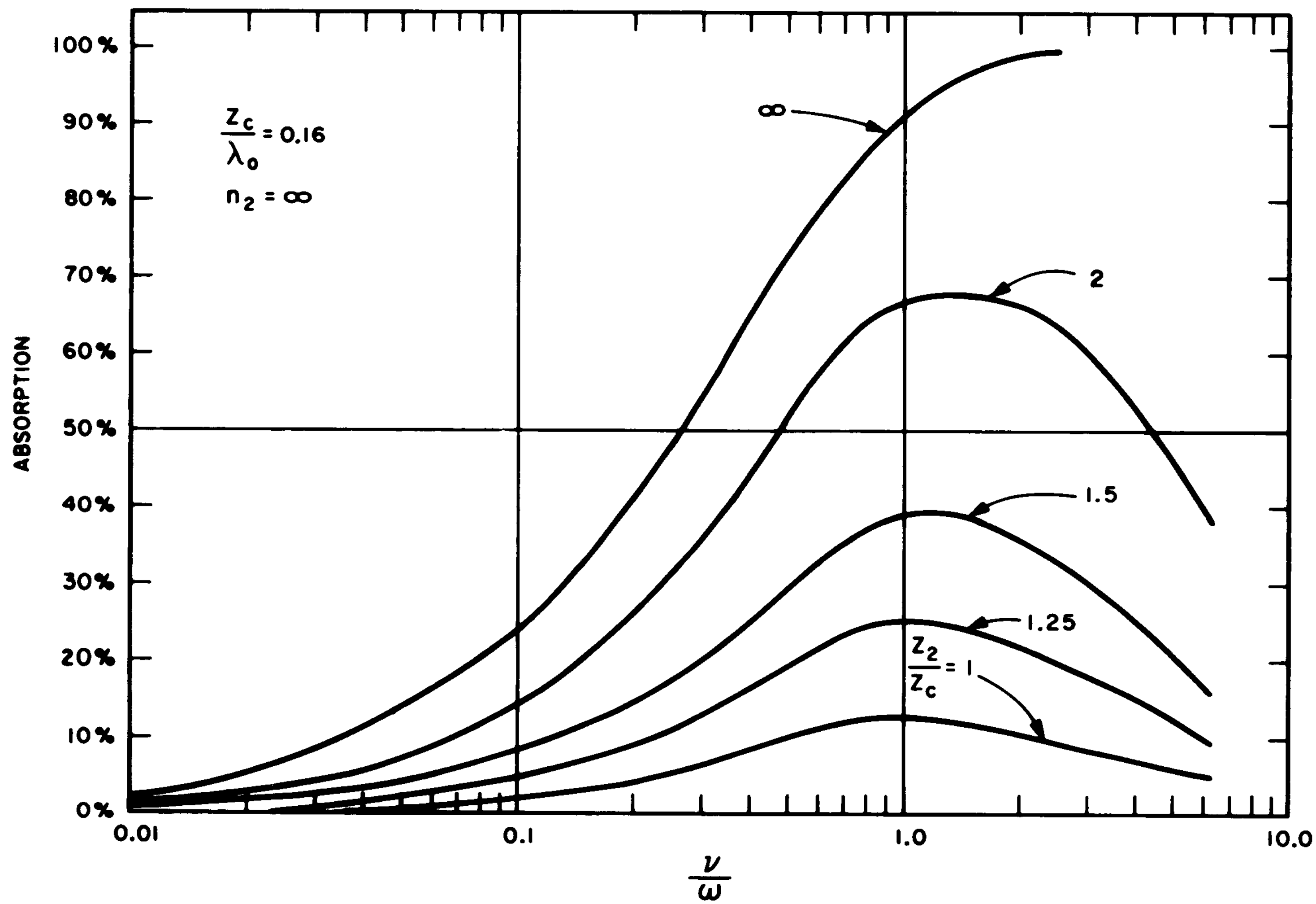

Figure 8f Total absorption for the Case IIIa

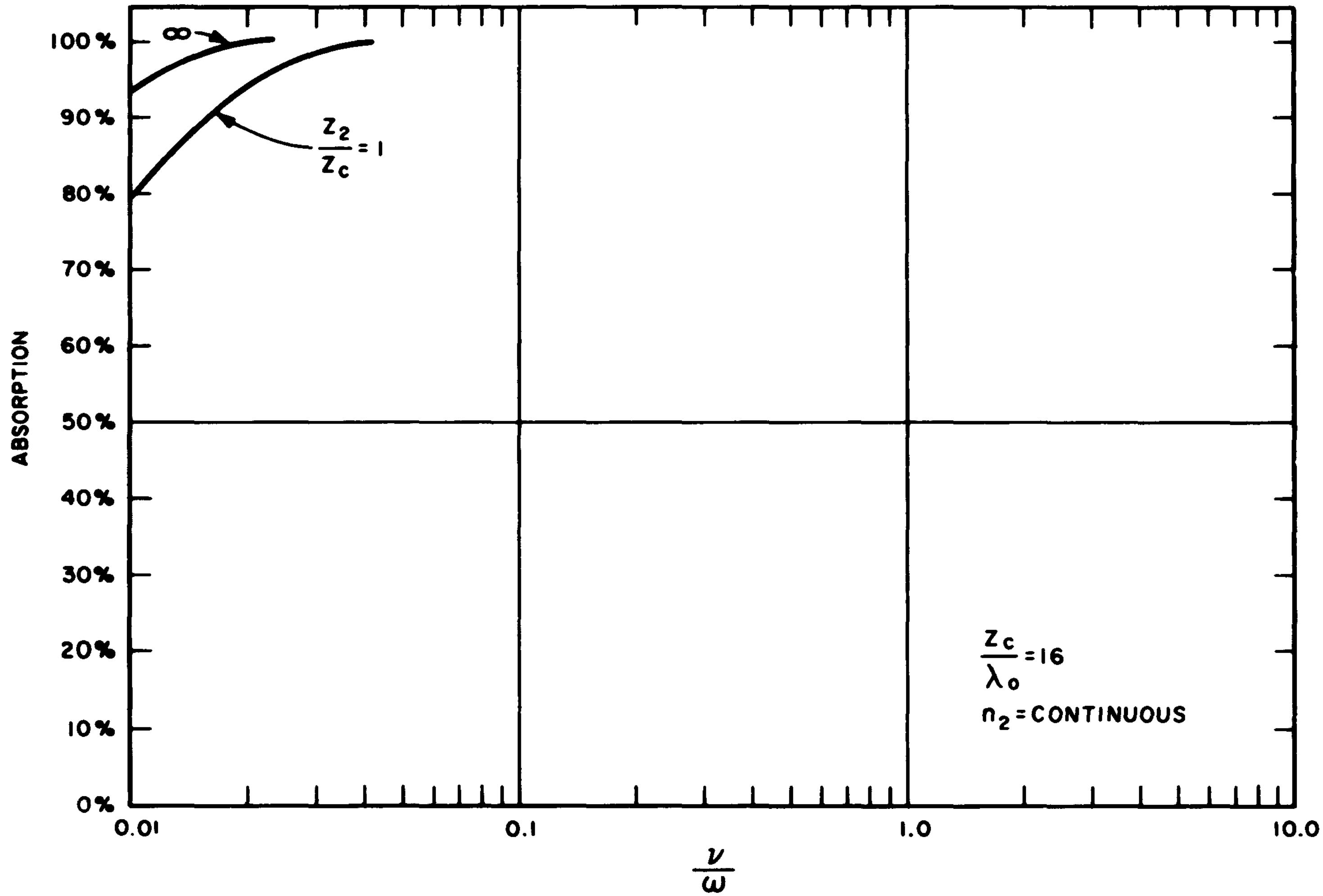

Figure 9a Total absorption for the Case IIIb

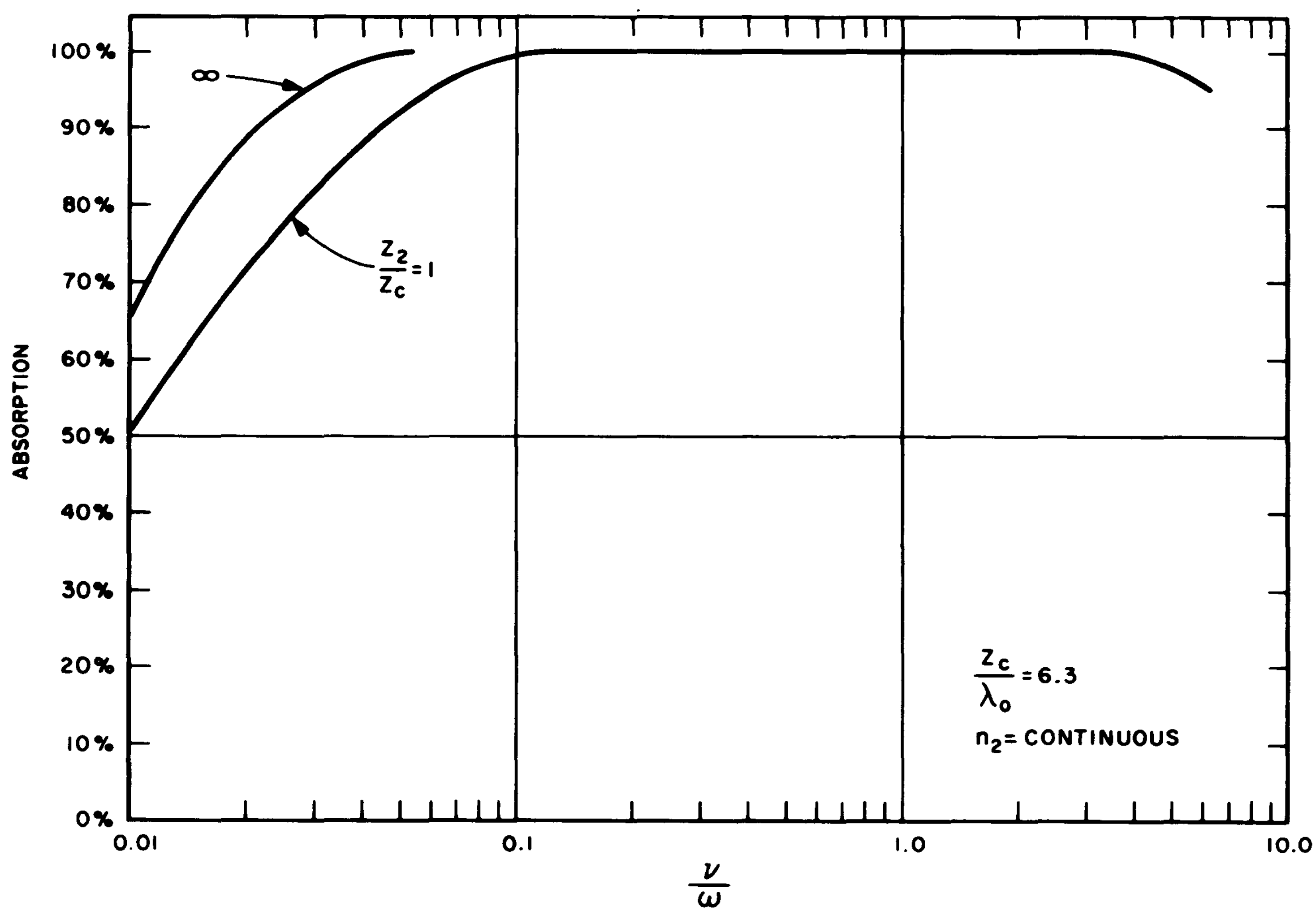

Figure 9b Total absorption for the Case IIIb

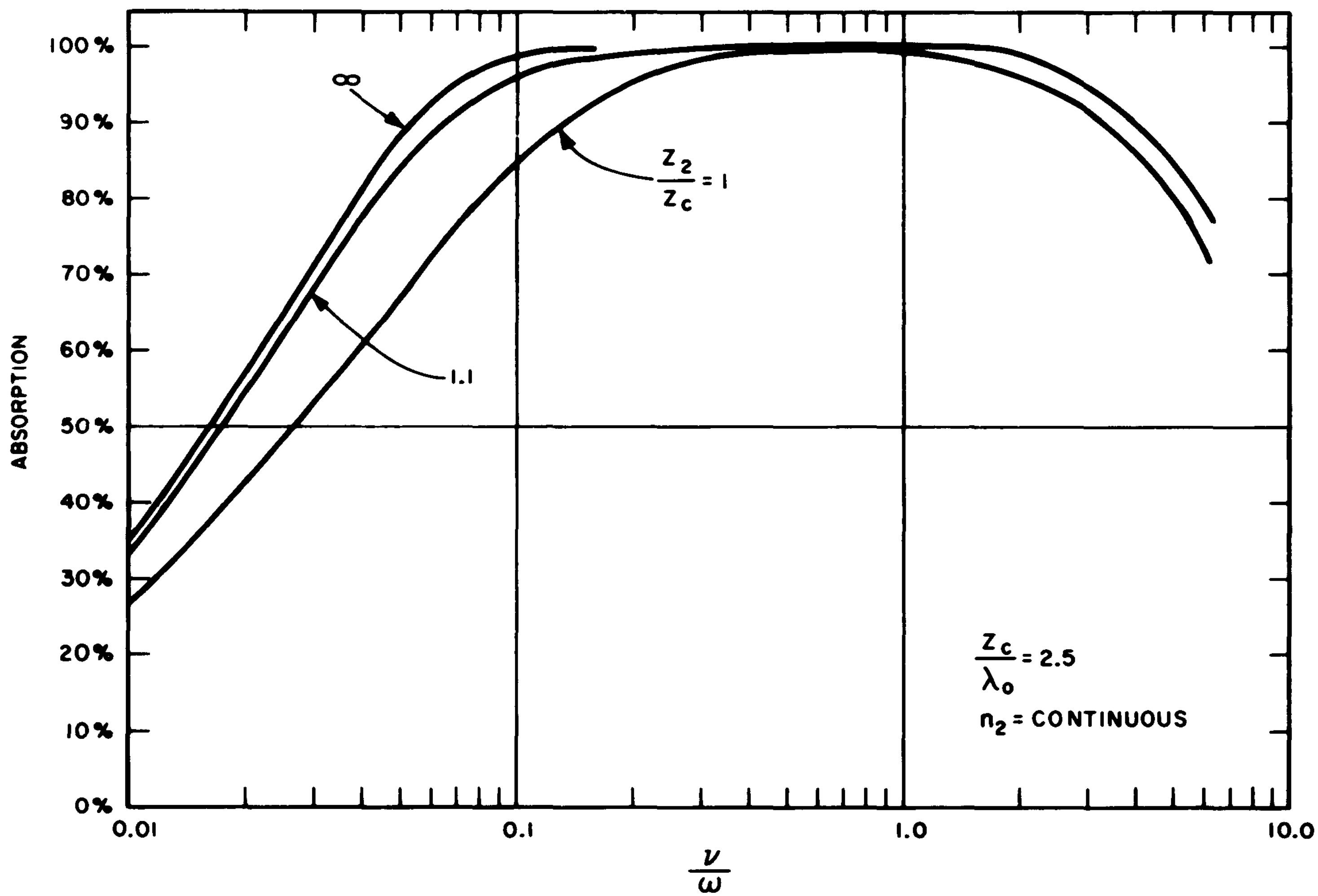

Figure 9c Total absorption for the Case IIIb

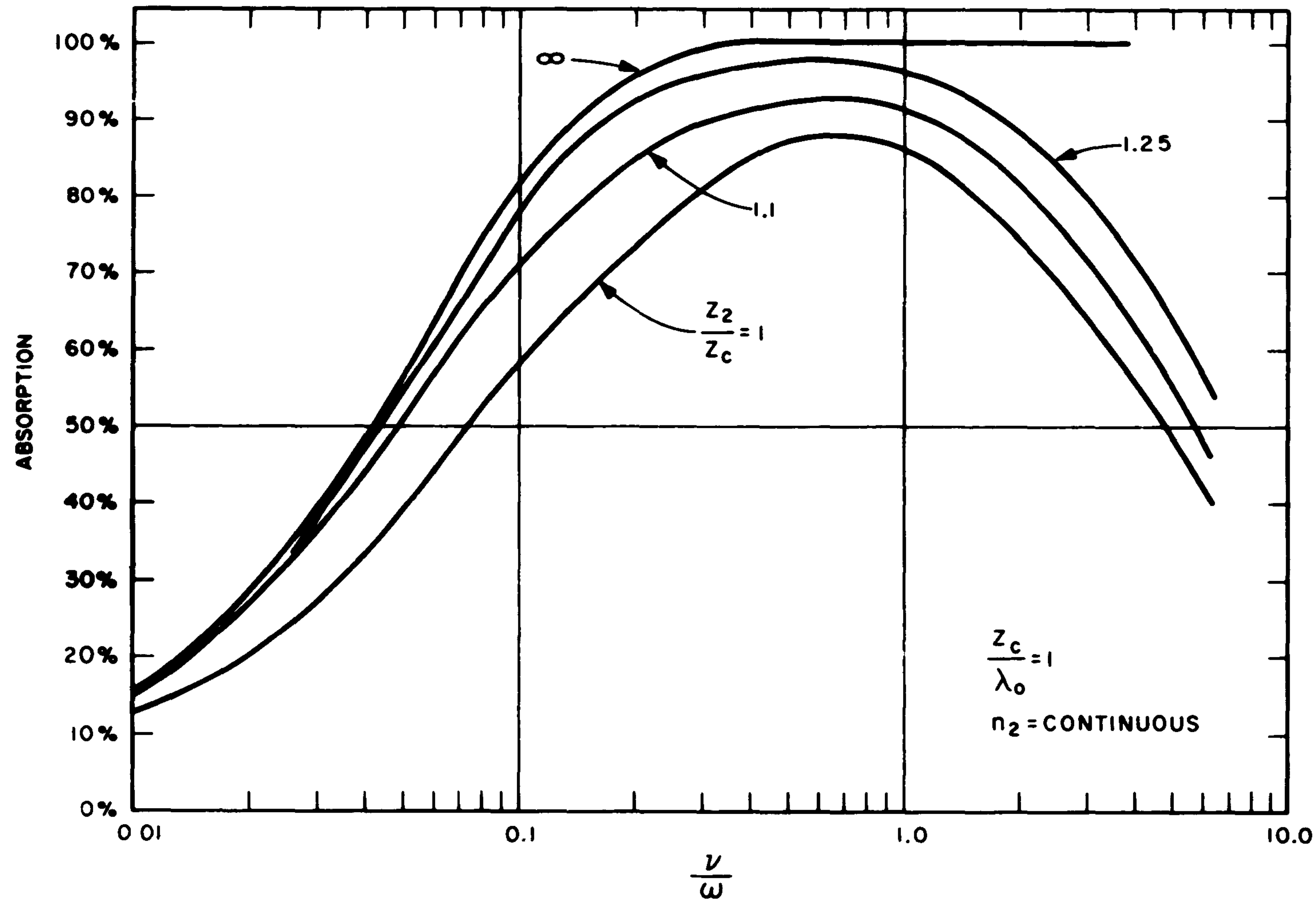

Figure 9d Total absorption for the Case IIIb

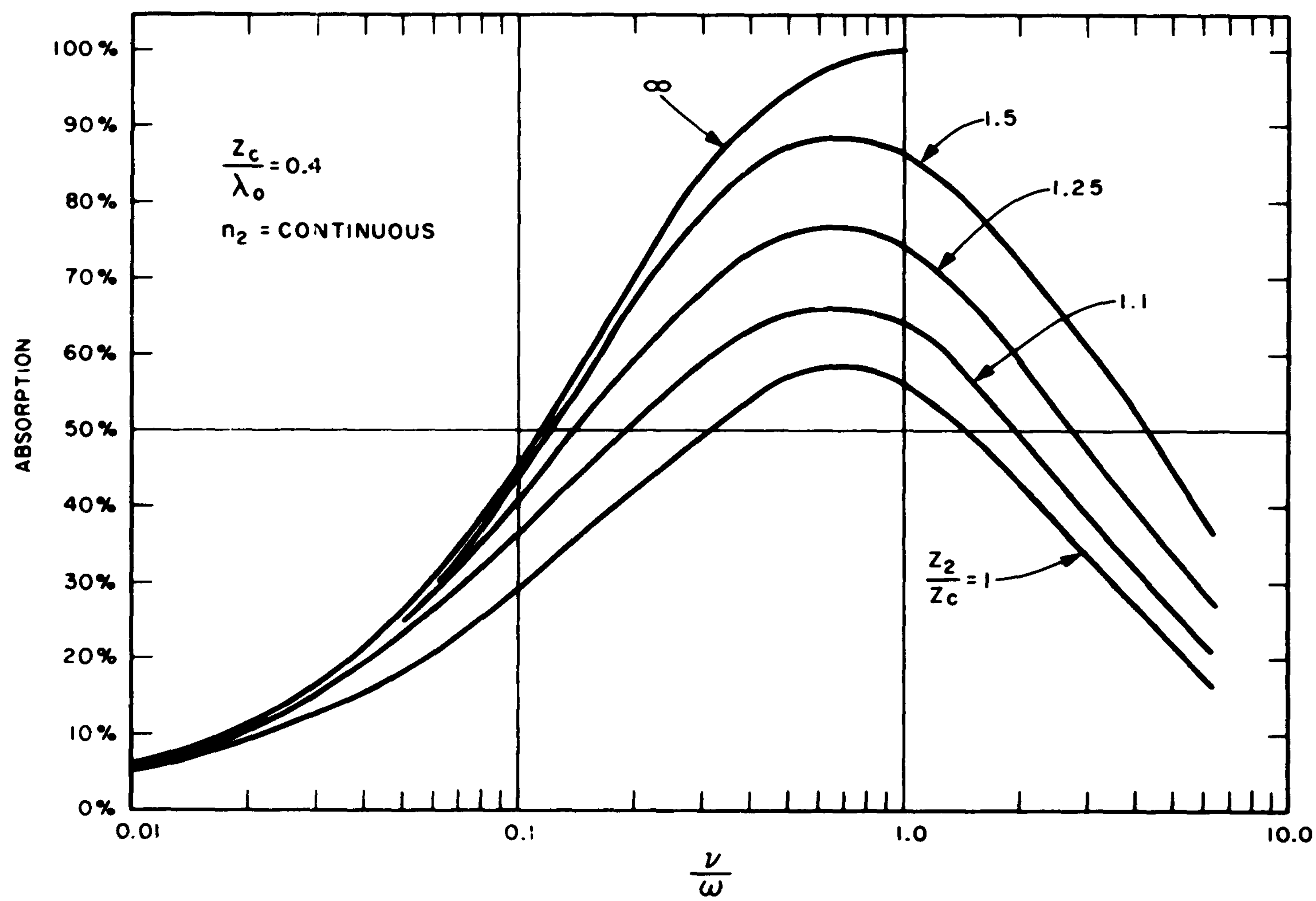

Figure 9e Total absorption for the Case IIIb

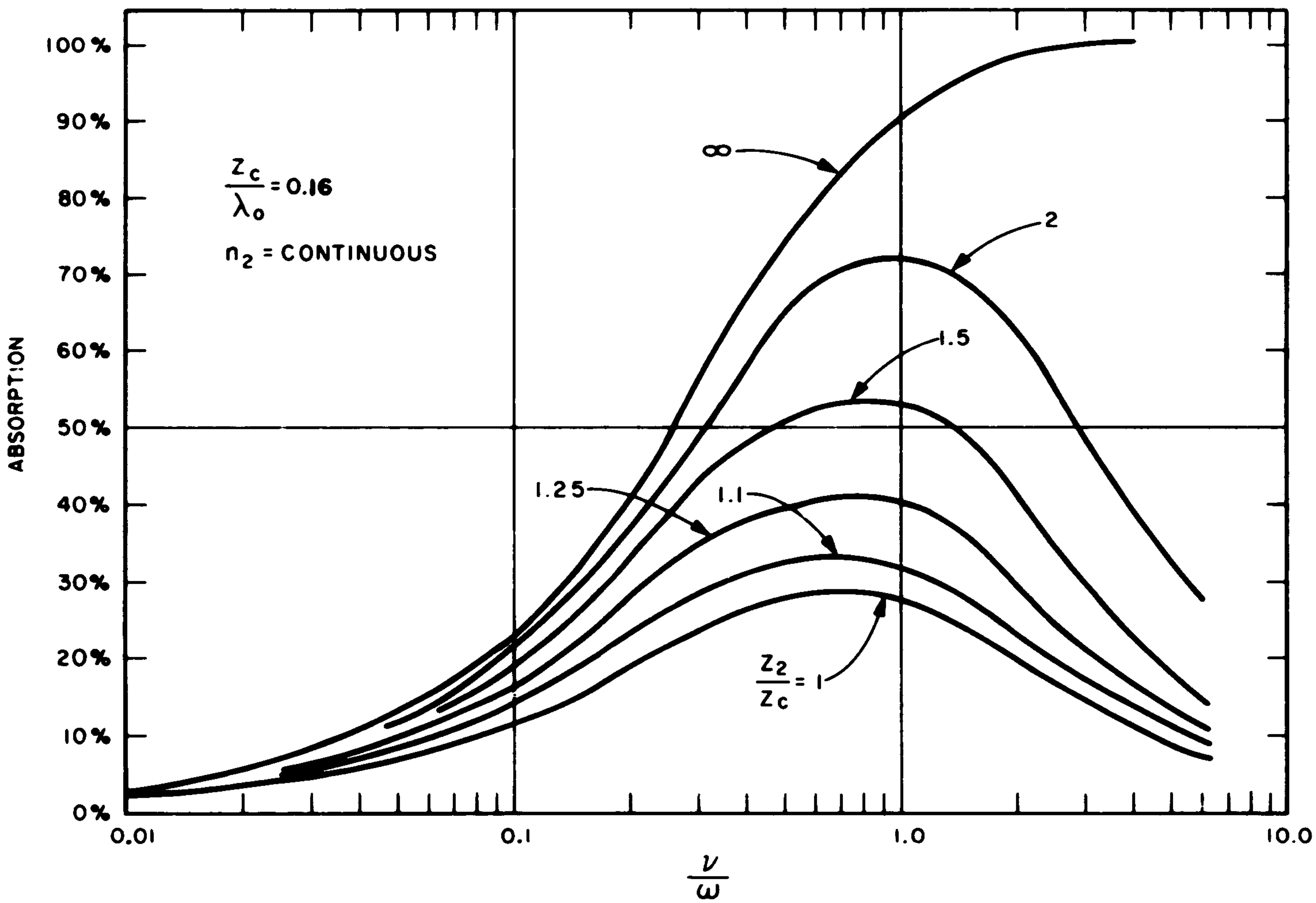

Figure 9f Total absorption for the Case IIIb

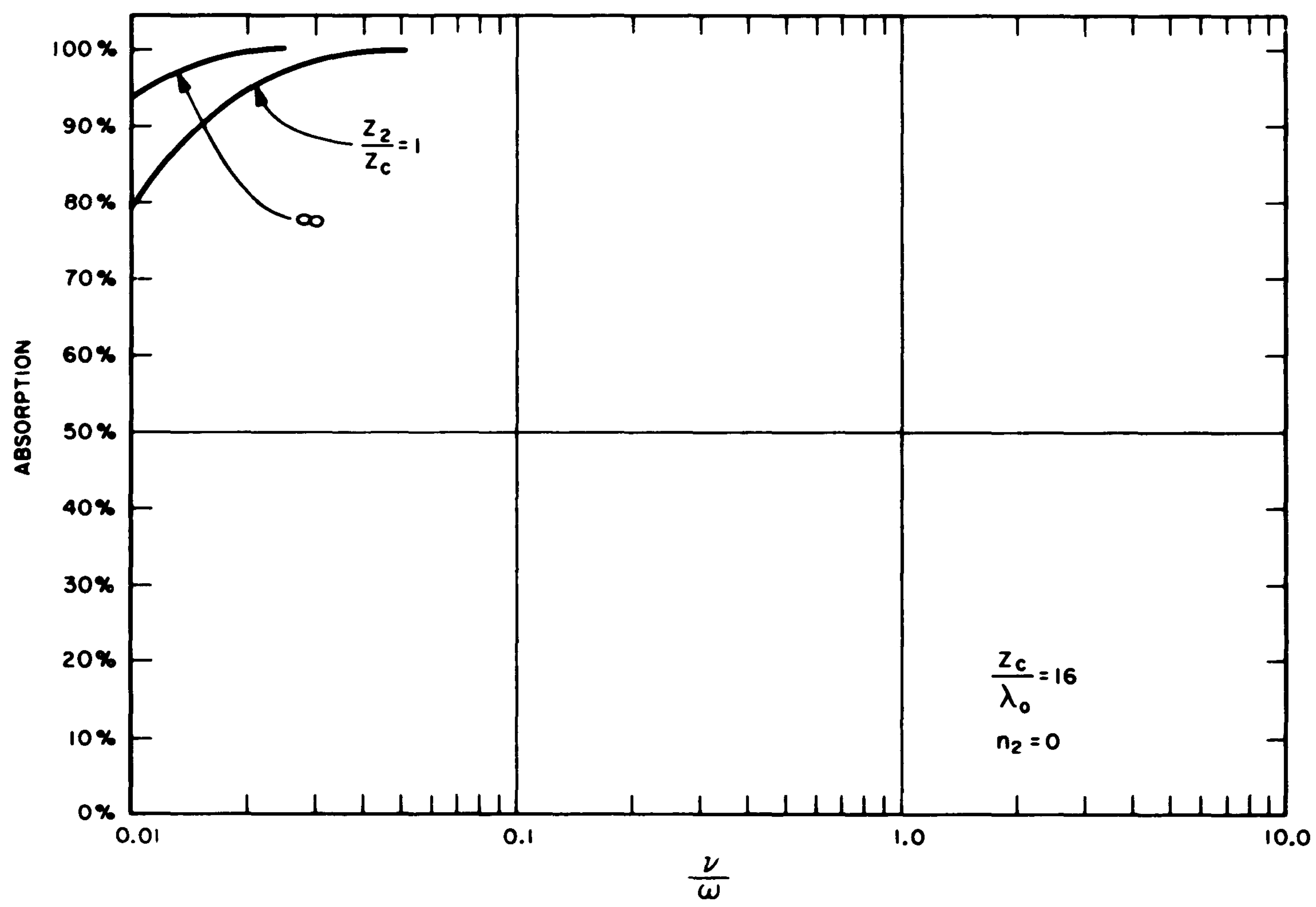

Figure 10a Total absorption for the Case IIIc

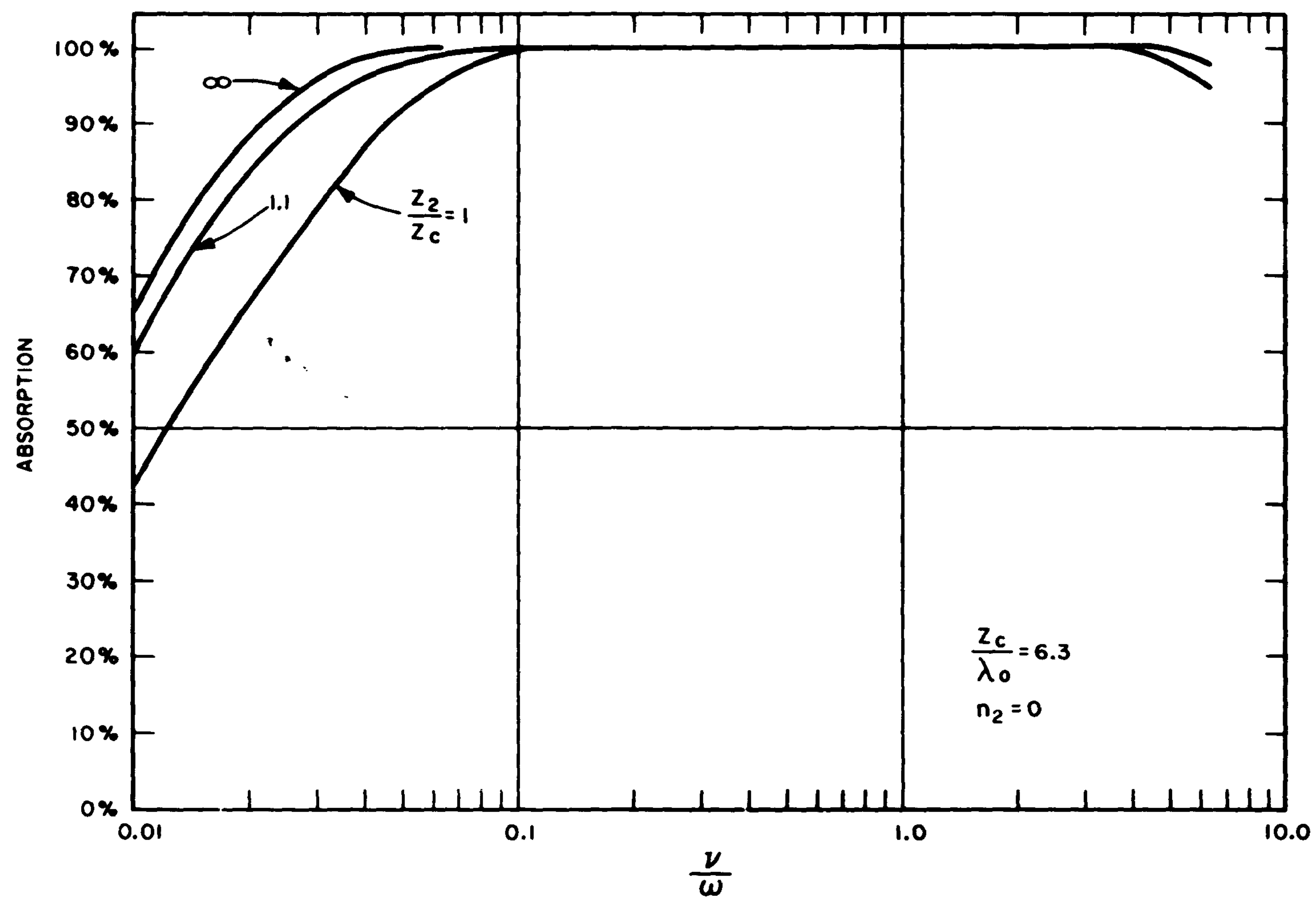

Figure 10b Total absorption for the Case IIIc

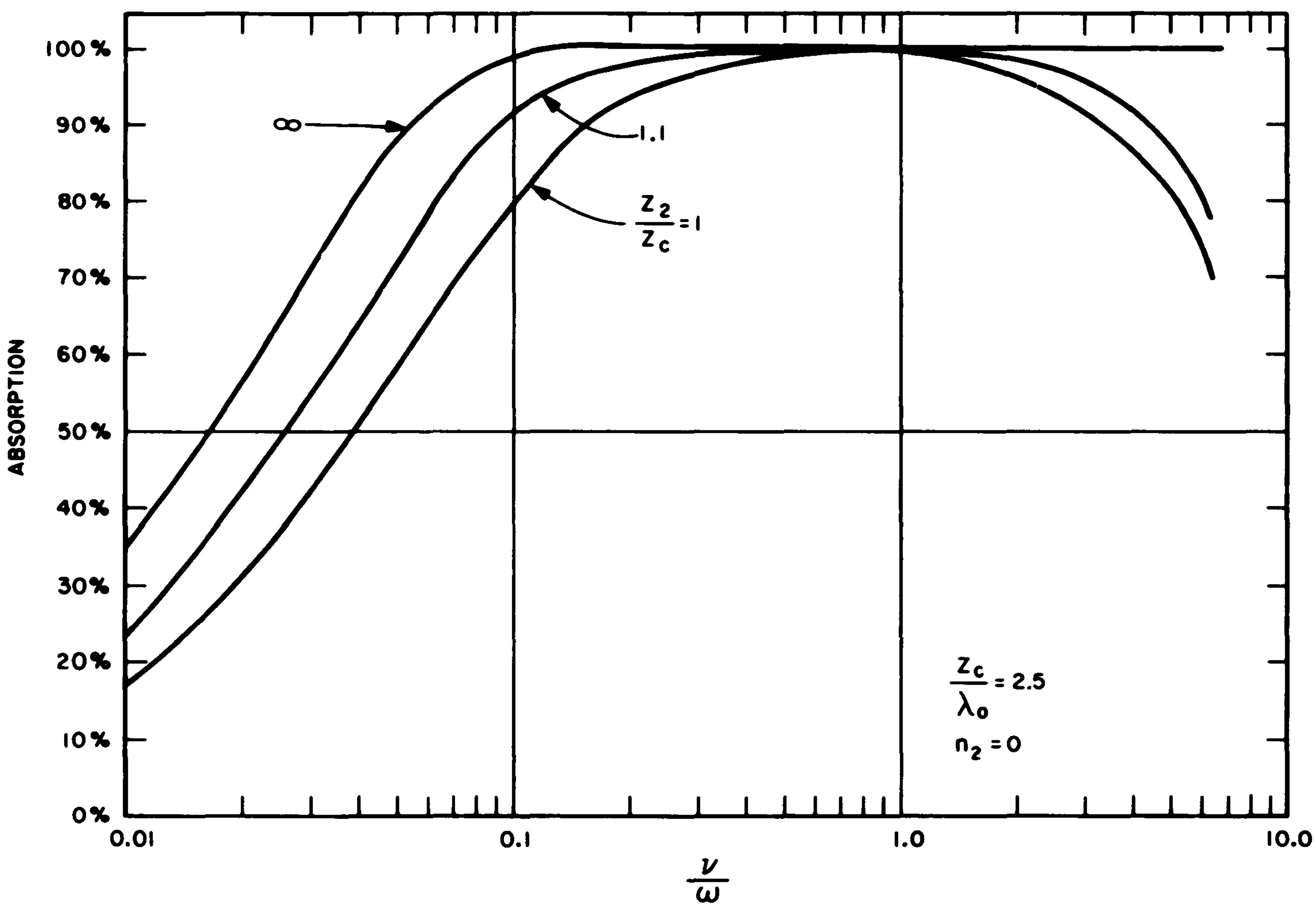

Figure 10c Total absorption for the Case IIIc

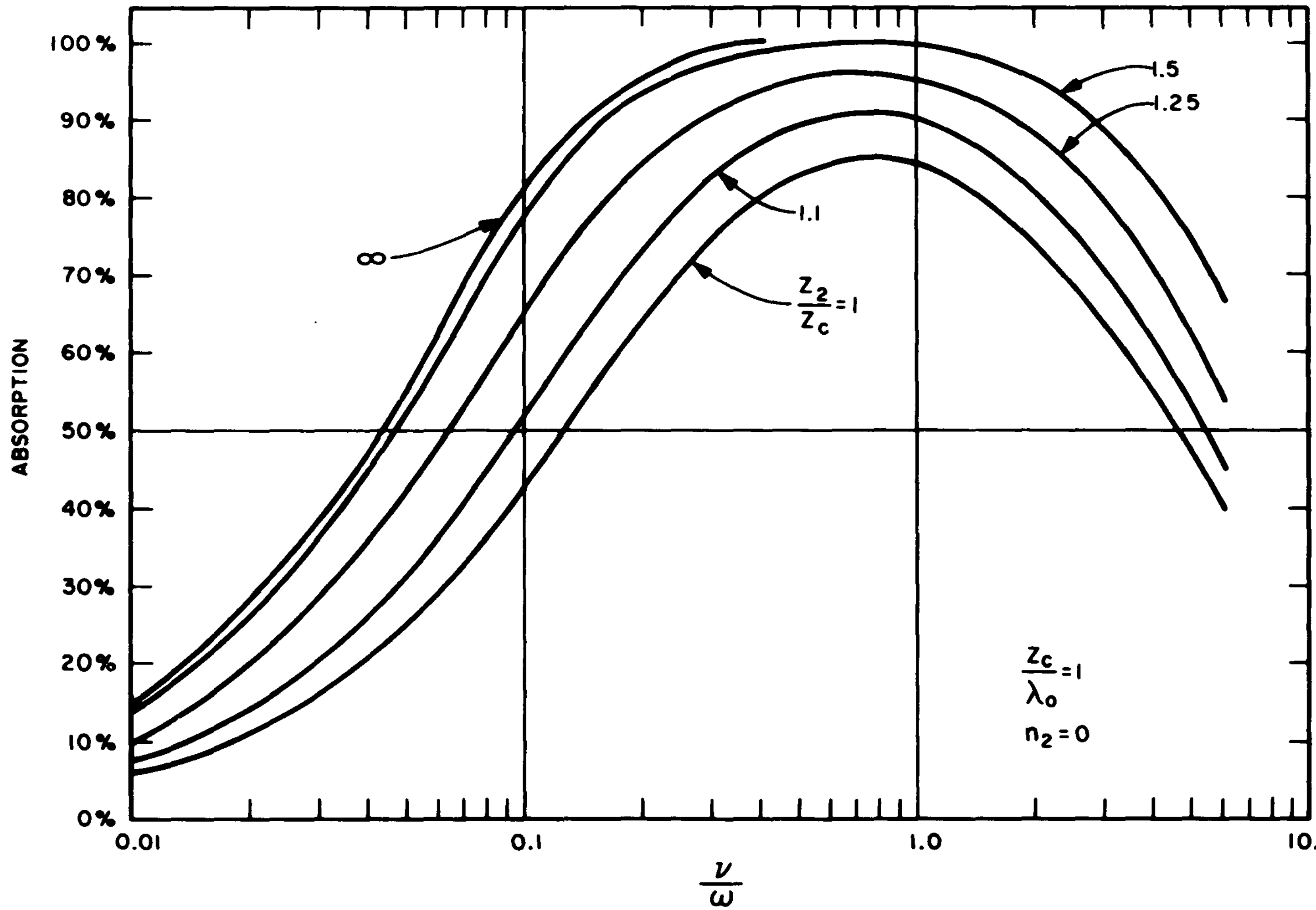

Figure 10d Total absorption for the Case IIIc

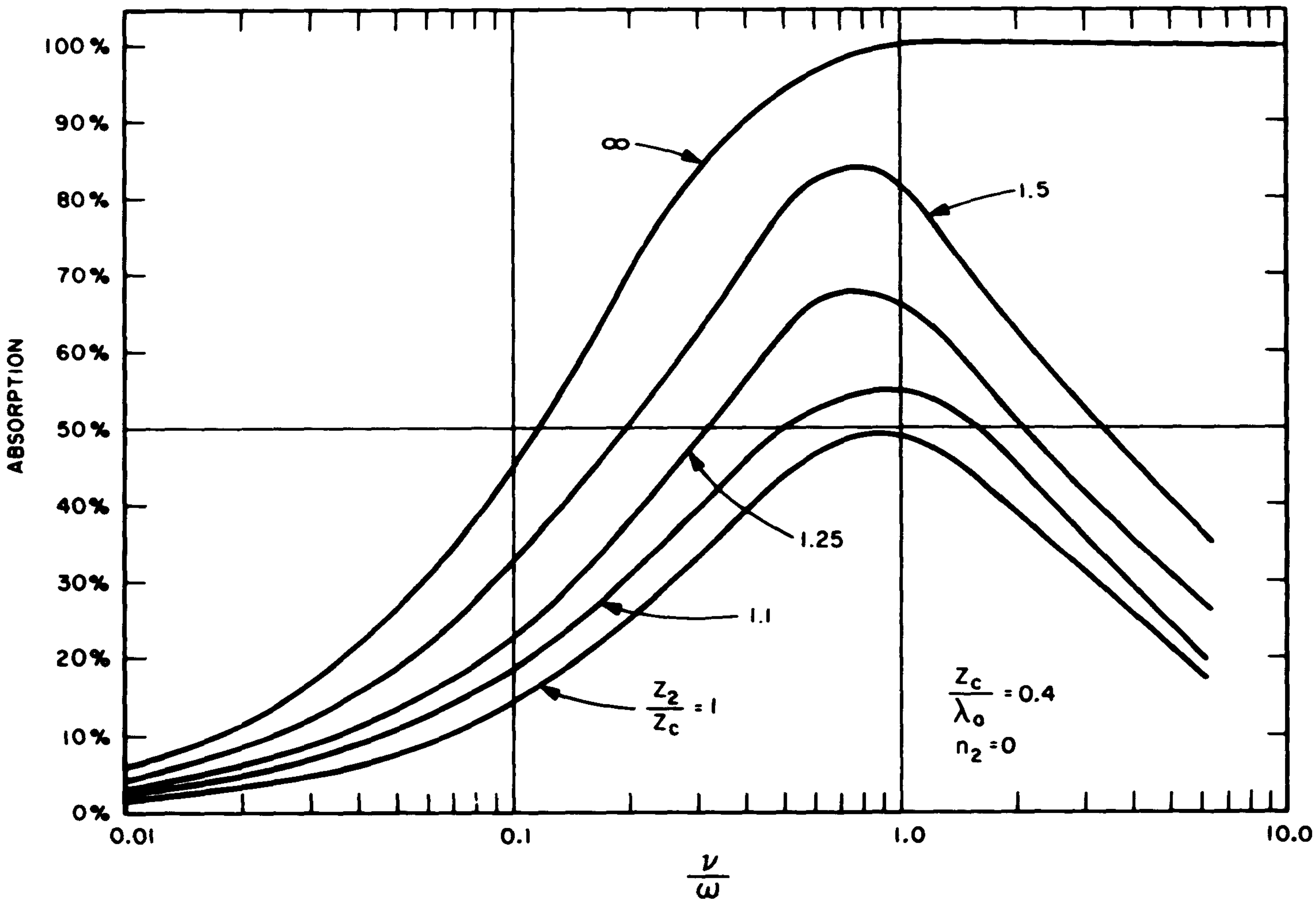

Figure 10e Total absorption for the Case IIIc

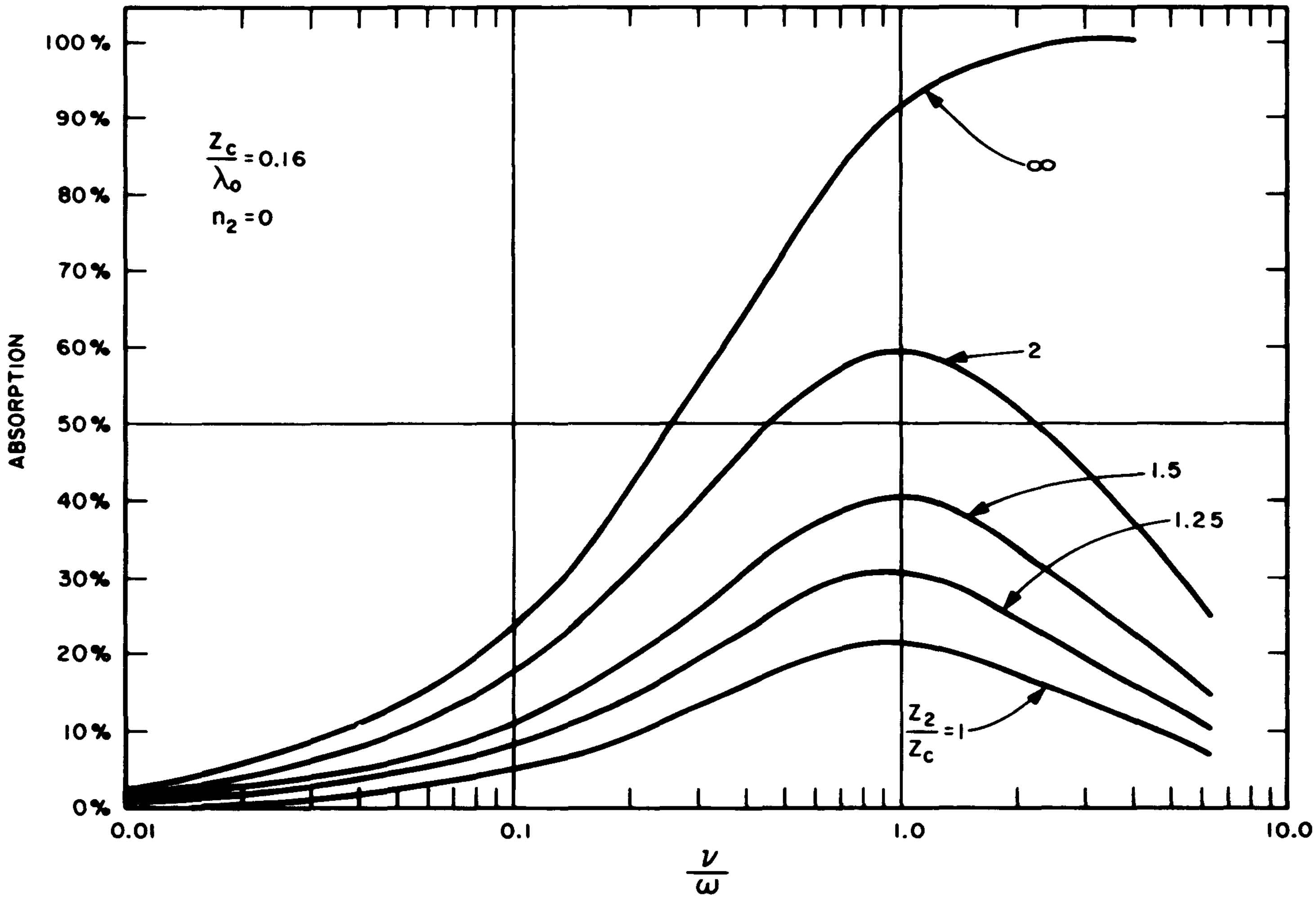

Figure 10f Total absorption for the Case IIIc

plotting ν/ω versus z_c/λ_o, with the absorption as a curve parameter. A surprising aspect of these curves is the fact that for values of $z_c/\lambda_o > 0.5$ they all consist of approximately straight lines with a slope of 45^o. Since the scale is logarithmic, this means that a given increase in z_c/λ_o is equivalent to a similar increase in the value of ν/ω. In fact, it is possible to combine all of these curves into a single curve which plots the absorption versus the product (z_c/λ_o) (ν/ω) as shown in Fig. 4. This curve holds with good accuracy provided z_c is larger than half a free-space wavelength. A single curve thus provides complete information over a wide range of parameters.

It is interesting to note that the absorption is substantial even for relatively steep gradients of the electron density. For example, with $\nu/\omega = 0.1$, the absorption equals 60 percent of the incident power for $z_c = \lambda_o/2$ and 30 percent for $z_c = \lambda_o/4$.

Figs. 5a-5g display the absorption W as function of distance. W is defined in Eq. (11) and is normalized in such a way that the integral under each curve equals unity. The intention is to compare the spacial distribution of the absorbed energy and not the absolute values of the absorption.

For a given value of z_c/λ_o and very small values of ν/ω, the curves are independent of ν/ω. This can be explained as follows. When ν/ω is very small, the absorption has a negligible effect on the field. The field distribution is therefore essentially the same as for $\nu/\omega = 0$. W is proportional to $|E|^2 \sigma$ where σ, the (real) conductivity, is

proportional to n and hence to z. Thus for any given ν/ω, W is proportional to $|E|^2 z$ and therefore has the same distribution for all small values of ν/ω. It therefore suffices to display a curve for $\nu/\omega = 0.01$ and then proceed directly to a value of ν/ω which produces a substantial deviation in W, omitting all intermediate values.

The general behavior can be characterized as follows. Consider first large values of z_c/λ_o (e.g., Fig. 5a). For small ν/ω there is a marked standing wave pattern, peaked near cut-off. As ν/ω is increased, considerable energy is absorbed before it reaches the cut-off region. Hence the reflected wave and with it the standing wave pattern disappears, and the absorption peak shifts to the left. For $\nu/\omega > 1$ the trend is reversed. For large ν/ω, the plasma is no longer characterized as a dielectric, but rather as a conductor whose resistivity increases with ν/ω. As ν/ω is increased, the radiation penetrates farther into the plasma.

For smaller values of z_c/λ_o (e.g., Fig. 5d) there is no longer a marked standing wave pattern. There is a single absorption peak which for small ν/ω is somewhat to the left of z_c. This peak shifts first to the left and then, for $\nu/\omega > 1$, back to the right, with increasing ν/ω.

For very small values of z_c/λ_o (Fig. 5g), the peak of the absorption is always considerably beyond the cut-off point and tends to shift to the right with increased values of ν/ω.

In the most interesting cases, the absorption thus seems to

be concentrated in a region somewhat in front of the cut-off point. There are, however, exceptions to this which should be borne in mind. These occur for $\nu/\omega > 1$, and for very large z_c/λ_o.

If one may assume that the most critical part of the medium is that in which most absorption occurs, then the decisive parameters in this problem are given by the average density gradient and the collision frequency in the region immediately in front of the cut-off point.

CASE II:

Figs. 6 and 7 represent respectively the two types of boundary conditions, given as Case IIa and Case IIb in Fig. 1. In the first, there is a continuous match at z_1 from a medium of density n_1 into the linear region. In the second, there is an abrupt jump from $n_1 = 0$ to the value which n assumes at z_1 in the linear medium. As compared to Case I, one would expect an improvement in the match in Case IIa and a worsening of the match in Case IIb. These effects are masked somewhat by an additional effect resulting from the fact that the introduction of a boundary at z_1 removes from consideration a section between $z = 0$ and z_1, in which some energy is absorbed and therefore tends to lower the value obtained for the total absorption. The latter effect is most pronounced for high values of z_c/λ_o. A study of Fig. 5 indeed shows that for large z_c/λ_o a large fraction of the absorption occurs far in advance of the cut-off point.

If we compare the curves corresponding to $z_1/z_c = 0.5$ to the reference curves corresponding to $z_1/z_c = 0$, we find that the difference

in absorption is less than 20 percent over much of the range of values (wherever the $z_1/z_c = 0.5$ curve is omitted, this indicates close agreement with the corresponding $z_1/z_c = 0$ curve).

The deviation becomes larger if z_1 approaches z_c more closely. For $z_1/z_c = 0.75$ it reaches 50 percent at some extreme points, but remains considerably below this level over most of the range. For $z_1/z_c = 0.9$, the deviation in most cases is even larger. The latter value represents, of course, an extreme perturbation of the original configuration, as it introduces the boundary right into the region of maximum absorption.

One may therefore conclude that fairly drastic changes in the density profile will have relatively little effect on the total absorption provided these changes do not occur too close to the cut-off point.

CASE III:

Figs. 8, 9, and 10 represent, respectively, the three types of boundary conditions given as Case IIIa, IIIb, and IIIc in Fig. 1. In all of these a boundary is introduced at a point $z = z_2$, beyond which the plasma is assumed to be of uniform density n_2. In Case IIIa, $n_2 = \infty$. This corresponds to complete reflection at the boundary. In Case IIIb, n_2 is chosen so as to give a continuous transition at z_2. In Case IIIc, $n_2 = 0$. In the last two cases, tunneling of radiation gives rise to a transmitted signal beyond $z = z_2$.

For all boundary conditions, there is a diminution of absorption as compared to Case I. This is probably caused by removing from

consideration the energy absorbed in the region $z > z_2$. On the whole, the effect of the boundary seems to depend more on its position, that is, on z_2/z_c than on the value of n_2.

The deviations from the curves $z_2 = \infty$, which correspond to the Case I situation, depend on the values of the parameters ν/ω and z_c/λ_o. In the range $\nu/\omega < 1$ and $z_c/\lambda_o > 0.4$ the deviation is fairly small, confined to within about 15 percent for $z_2/z_c = 1.5$ and 30 percent for $z_2/z_c = 1.25$. The situation corresponding to $\nu/\omega > 1$ or $z_c/\lambda_o < 0.4$ can be understood by considering the spacial distribution of the absorption W as displayed in Fig. 5. There it is seen that in the above ranges the radiation penetrates very deeply beyond the cut-off point, and the absorption is therefore quite sensitive to any changes in this region.

Generalizations from the linear density variation to more general types of variation is therefore applicable only in the range $z_c/\lambda_o > 0.4$ and $\nu/\omega < 1$.

V. SUMMARY

In order to obtain a simple but general method for determining in a rough manner the absorption of microwave radiation incident on a plasma of tapered electron density, a calculation was performed for the special case of a linear density variation. Such semi-infinite linearly varying plasmas are characterized by the two parameters ν/ω and z_c/λ_o. The value of the total absorption is displayed in Figs. 2 and 3. It turns out that over a very wide range of parameter values the absorption

depends only on the combined parameter (z_c/λ_o) (ν/ω) thus making it possible to summarize a vast amount of data within the single curve represented in Fig. 4; Fig. 5 represents the distribution W of this absorption along the propagation axis.

In order to determine to what extent the absorption depends solely on conditions in the neighborhood of cut-off, the semi-infinite plasma was modified by imposing several types of boundary conditions at points in front and behind the cut-off point. These modifications are represented schematically in Fig. 1, and the resulting changes in absorption are tabulated in Figs. 6, 7, 8, 9 and 10. From these one can conclude that in the range $\nu/\omega < 1$ and $z_c/\lambda_o > 0.4$, the deviations are not too severe. One may therefore conclude that in this range of parameters the absorption in a semi-infinite plasma of linear density variation as represented by the data on Case I is representative of any inhomogeneous plasmas whose density at the boundary rises reasonably smoothly from zero to a value above cut-off. Such a plasma would be characterized by the gradient of the density at the cut-off point, and by ν/ω in the same neighborhood. Even outside the above range, the data will give a general indication of the magnitude of the absorption.

The accuracy of such a procedure will, of course, depend on the particular profile studied. The inaccuracies are, however, small when compared to those involved in the determination of the experimental conditions actually prevailing in a practical situation.

VI. BIBLIOGRAPHY

1. See e.g. British Association Mathematical Tables, Part-Vol. B, The Airy Integral.